Uncertainty of flow measurements – examples

Principle considerations

flow meter

sources of measurement uncertainties

- repeatability
- resolution
- dynamische Eigenschaften wie Zeitkonstante
- etc

expanded uncertainty of the meter to be calibrated

$$U = k\sqrt{u_z^2 + u_A^2}$$

test rig

sources of measurement uncertainties

- influence of measurement principles (start-stop, dynamic, balance, container)
- temperature
- pressure
- viskosity
- density
- absolute humidity
- etc.

What do you mean by flow rate Q?

volume flow rate Q_V

$$Q_V = \frac{dV}{d\tau} = \dot{V}$$

mass flow rate Q_m

$$Q_m = \frac{dm}{d\tau} = \dot{m}$$

τ... time

In most cases we ask for the volume V resp. the mass m

- it is difficult to keep the flow rate constant

- dynamic flow rate measurement: integral of the actual value of flow rate over time ➔ result: mean flow rate

- In the following we investigate the dynamic flow measurement: we measure the volume *V* or the mass *m* and define the flow rate as volume divided by time

Example 1:

Measurement of gas flow

Exanple 1: model equation

Basis: ideal gas equation: $pV = \nu R_G T$

$$\boxed{V = \frac{\upsilon R_G T}{p} = V(\upsilon, R_G, T, p)}$$

= **model equation**

input quantities:

- mol number υ,
- gas constant R_G,
- temperature T
- pressure p

reference to standard condition: index N, operational conditions: index B**:**

$$V_N = V_B \frac{T_N \, p_B}{T_B \, p_N} = V(T_N, T_B, p_N, p_B)$$

Example 1: Combined variance

Simplification: $\boldsymbol{R_G}$ = natural constant .... $u^2(R_G) \approx 0 \Rightarrow$

$$u_c^2 = \left(\frac{\partial V_N}{\partial V_B}\right)^2 u^2(V_B) + \left(\frac{\partial V_N}{\partial T_N}\right)^2 u^2(T_N) + \left(\frac{\partial V_N}{\partial T_B}\right)^2 u^2(T_B) +$$

$$+ \left(\frac{\partial V_N}{\partial p_N}\right)^2 u^2(p_N) + \left(\frac{\partial V_N}{\partial p_B}\right)^2 u^2(p_B)$$

calculation of sensitivity coefficients: 1 $\Rightarrow$

$$\frac{u_c^2}{V_N^2} = \frac{u^2(V_B)}{V_B^2} + \cancel{\frac{u^2(T_N)}{T_N^2}} + \frac{u^2(T_B)^2}{T_B^2} + \cancel{\frac{u^2(p_N)}{p_N^2}} + \frac{u^2(p_B)}{p_B^2}$$

Example 1: real gas factor, conditional number

in reality: consideration of real gas factors $\Rightarrow$ ***z***

$$\boxed{pV = \upsilon R_G T\, z}$$

$$\boxed{V_N = V_B \frac{T_N\, p_B\, z_N}{T_B\, p_N\, z_B} = V(T_N, T_B, p_N, p_B, z_N, z_B) = V_B\, \boldsymbol{Z}}$$

Z = conditional number

$$\frac{u_c^2}{V_N^2} = \frac{u^2(V_B)}{V_B^2} + \cancel{\frac{u^2(T_N)}{T_N^2}} + \frac{u^2(T_B)^2}{T_B^2} + \cancel{\frac{u^2(p_N)}{p_N^2}} + \frac{u^2(p_B)}{p_B^2} + \cancel{\frac{u^2(z_N)}{z_N^2}} + \frac{u^2(z_B)}{z_B^2}$$

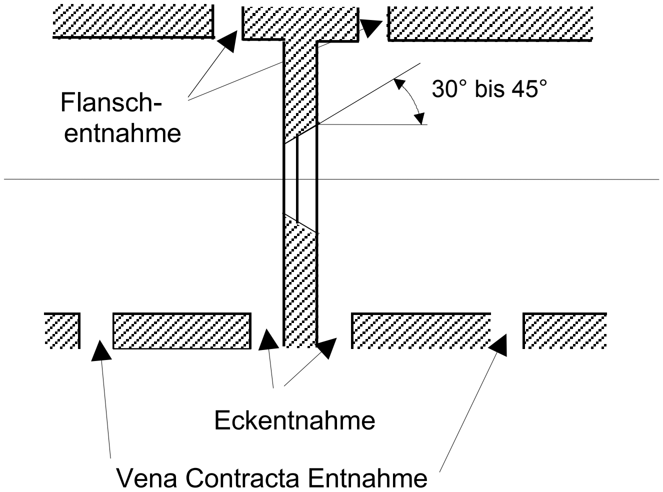
Example 1 – practical procedure

T_i, p_i and z_i known quantities

$u(T_i)$, $u(p_i)$ und $u(z_i)$ .. assessment by the known measurement process

Example 2:

Mass flow measurement with an orifice plate



quantity: pressure difference Δp →

$$\boxed{Q_m = \dot{m} = A\sqrt{2\,\Delta p\,\rho}}$$

model equation

A ... flow area (in example fix)
ρ ... density

$u(A) \approx 0$ → combined variance:

$$u_c^2 = \left(\frac{\partial Q_m}{\partial \Delta p}\right)^2 u^2(\Delta p) + \left(\frac{\partial Q_m}{\partial \rho}\right)^2 u^2(\rho)$$

density $\rho = f(t)$: → approach: $\rho = \rho_0 \left[1 + \alpha\,(t - t_0) + \beta(t - t_0)^2\right]$

t ... temperature and
t_0 ... reference temperature

example 2: of what order is $u(\Delta p)$?

$$u^2(\rho) = \left(\frac{\partial \rho}{\partial t}\right)^2 u^2(t)$$

Because: $\rho/\rho_0 \approx 1$ $\rightarrow$

$$\boxed{\frac{u_c^2}{Q_m^2} \approx \frac{1}{4}\left[\frac{u^2(\Delta p)}{(\Delta p)^2} + \alpha^2 u^2(t)\right]}$$

Numerical example: water as a flow carrier $\rightarrow$

$\rho = \rho_0\,(1+\alpha\, t + \beta\, t^2) \approx 1000\,(1 - 1{,}431.10^{-4}\, t - 2{,}89.10^{-6} t^2)$

maximum permissible errors: $\pm$ 1 % $\rightarrow$

Question: of what order is $u(\Delta p)$ maximally ?

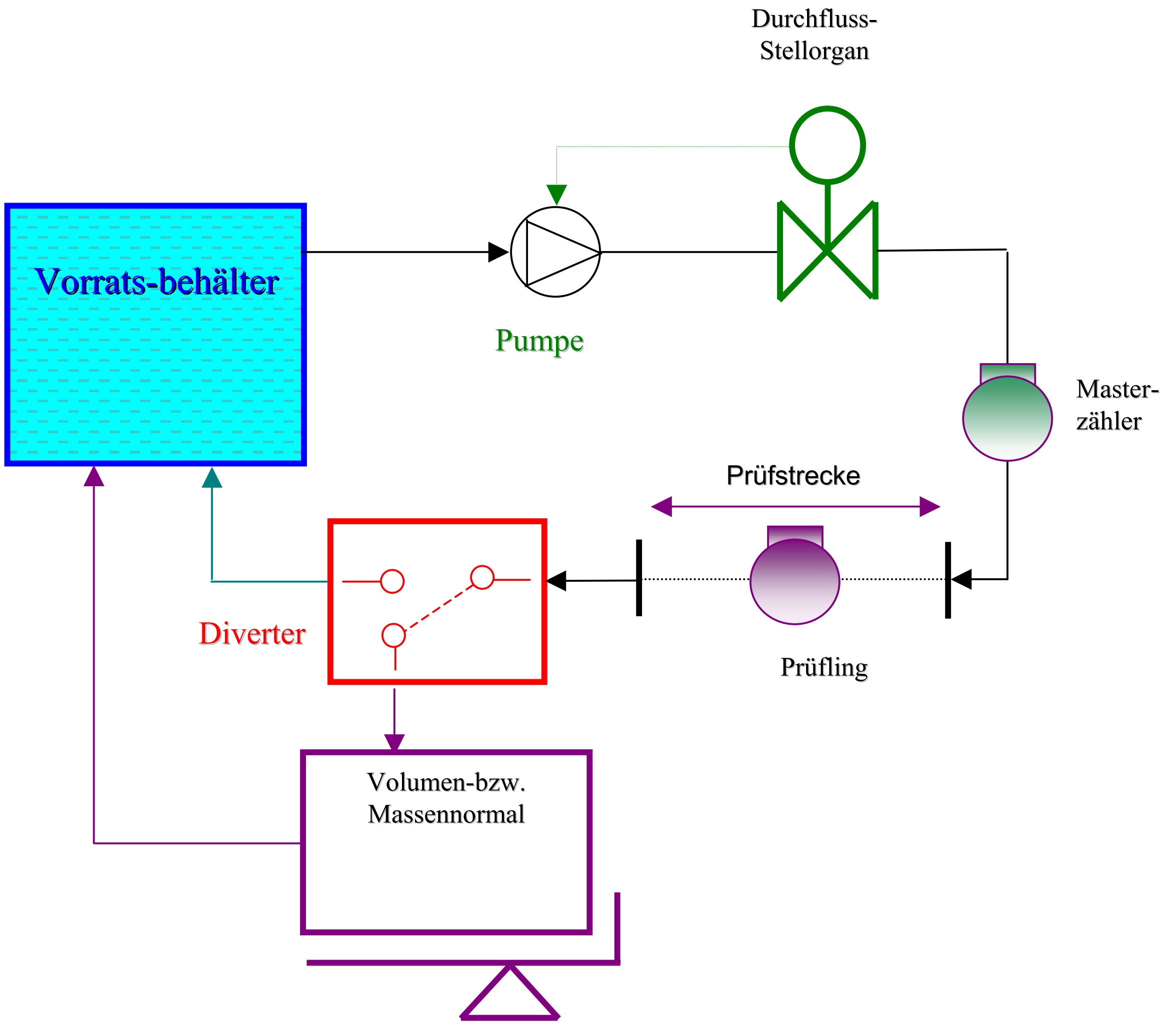
Answer: α and $u(t)$ is very small (Δt < 0,1 K) $\rightarrow$ maximum uncertainty is 0,5 % (u_c !) depends only from $u(\Delta p)/\Delta p \approx 1$ %

Example 3:

Calibration of a water meter at a test rig for hot water

View of a modern test rig for calibration of water meters

Working scheme



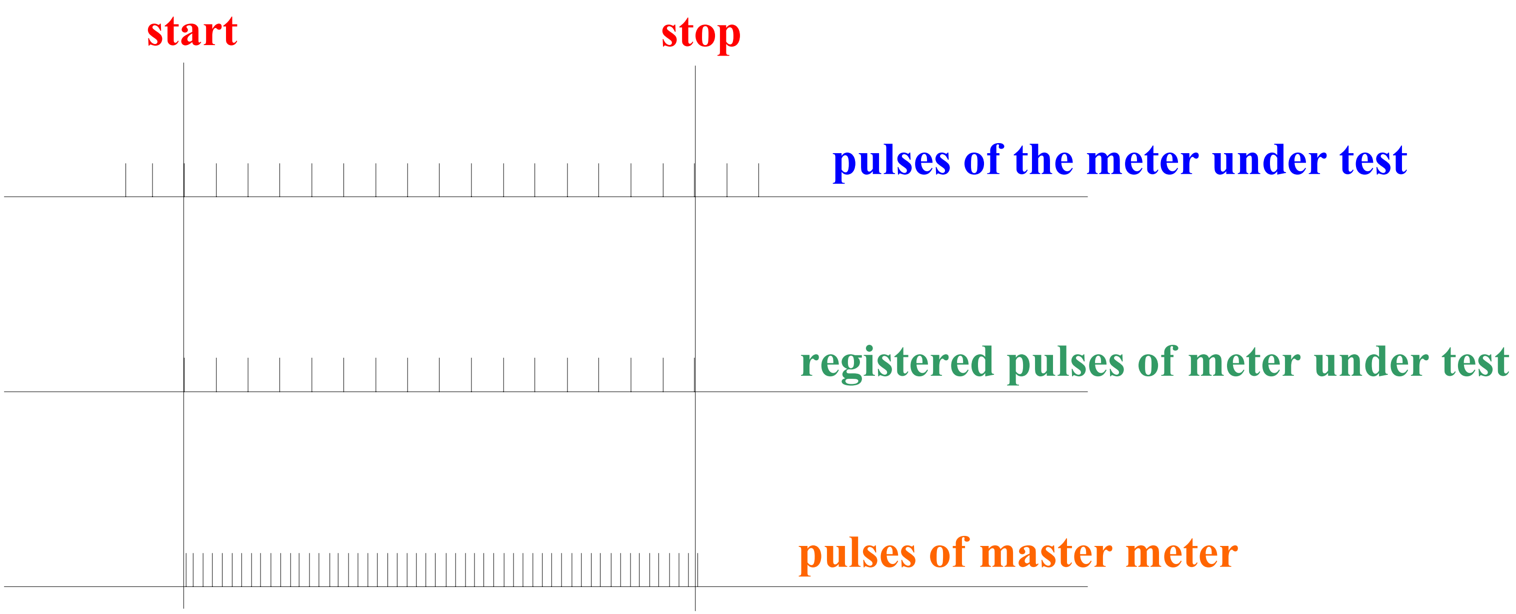
Calibrations of water meters and flow sensors of heat meters

- start-stop mode (exception)

- **dynamic mode**: meter under test is compared by a master meter. Master meter is calibrated at each measurement point by a balance
  simultaneous calibrations of several meters under test is possible

- comparison by master meter if weighing is not possible (pressurized mode)

control in dynamic mode



flow range

6 L/h $\leq$ Q $\leq$ 180.000 L/h

generation of flow by:

- pump fed by containers
- upper level containers (1,7 bar)

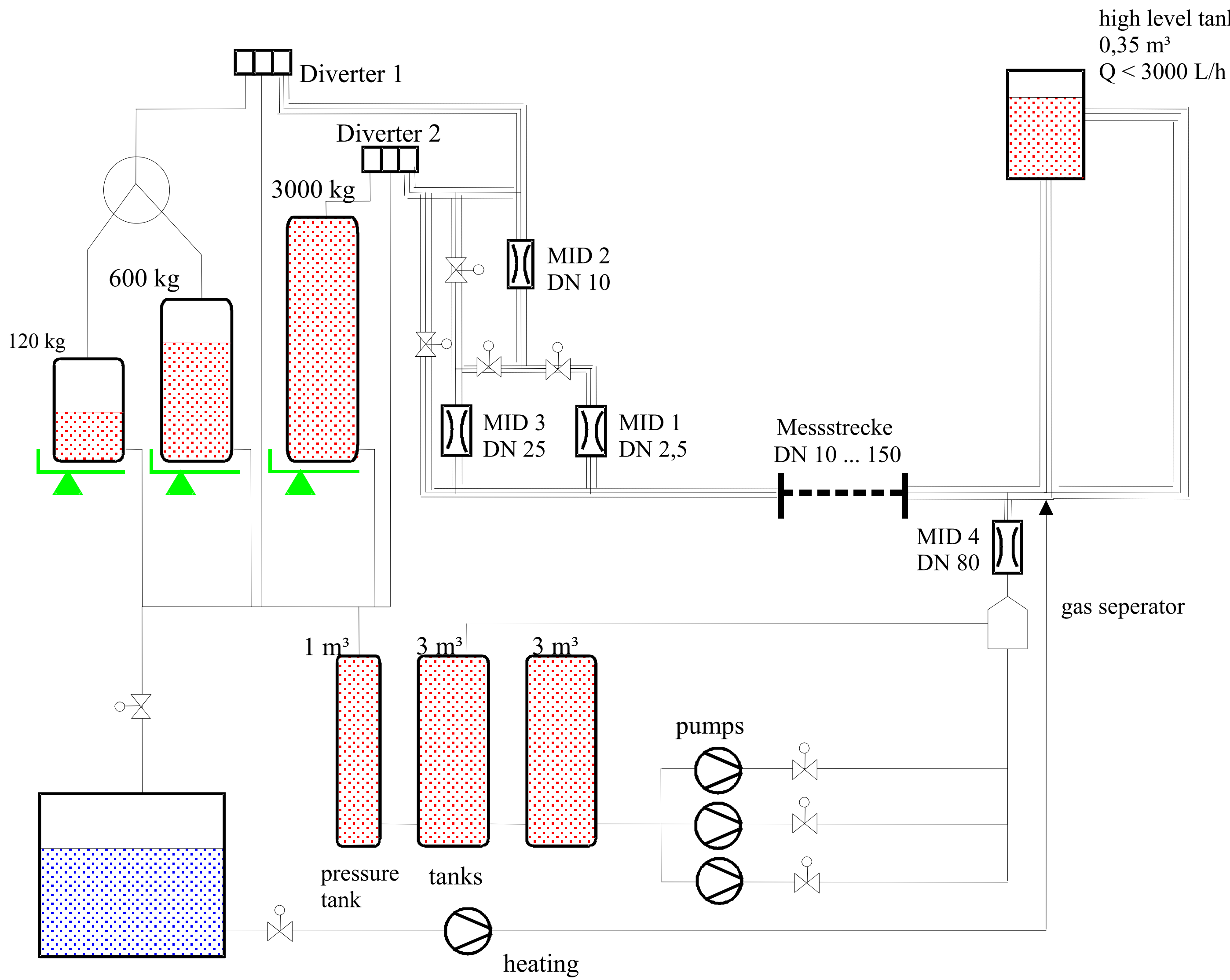
range of pressure:

5 bar

range of temperature:

- cold water: 2,5 °C $\leq t \leq$ 30 °C
- warm water: 30 °C $\leq t \leq$ 95 °C
- hot water: (pressurized mode) $t \leq$ 130 °C

Genaueres Schema des Prüfstandes



Sources:

- bouyancy, density
- evaporation
- systematic error of master meter
- diverter

Wanted: real volume

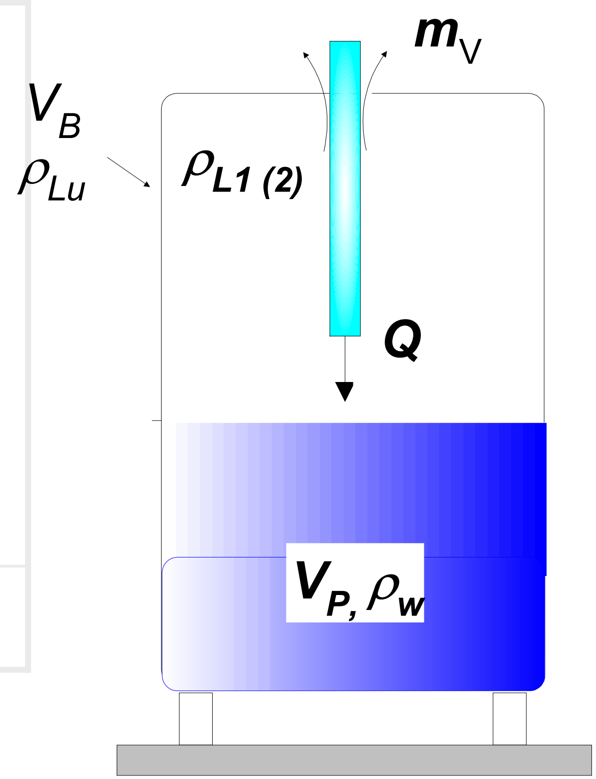
one measures: conventional value (= indicated value)
wanted: mass of water through the meter under test

conventional value ➜ mass =
f(air desity, humidity, density of water, evaporation):

$$m_N = \frac{(1-\frac{f_2}{\rho_W})\,\Delta W(1-\frac{\rho_{Lu}}{\rho_N}) - V_0\,(\rho_{L2}-\rho_{L1})}{(1-\frac{\rho_{L2}}{\rho_W})} - V_B\,(f_2 - f_1) + m_V$$

$$\boldsymbol{m_N \rightarrow V_N = m_N/\rho}$$

m_V	...	evaporation flow
f_1	...	absolute humidity in container before calibration
f_2	...	same, after
ρ_w	...	density of water used
ρ_{Lu}	...	density of air sourrounding the container
ρ_{L1}	...	density of air in the container before calibration
ρ_{L2}	...	same, after calibration
ρ_N	...	density of weights = 8000 kg/m³
V_B	...	container volume



evaporation flow:

$$m_V = \int_0^{\tau_P} f(\tau)\,\frac{dV_L}{d\tau}\,d\tau = \int_0^{\tau_P} f(\tau)\,Q\,d\tau$$

In principle: GUM

- systematic **errors are considered as they are known**

- **rest: looking for contributions to measurement uncertainty**

Procedure:

- Physically caused influence sources ➜
  type B, mostly rectangular oder similar

- repeatibility measurments whenever possible ➜ type A

- stabilities ➜ rectangular, trapezoid ....

sources of measurement uncertainty – overview - 1

- **resolution of meter under test** → pulse value

- **resolution of master meter** → pulse value

- **balance** (s) → variance from measurement errors plus short and long term stability

- **volume determination by weighing**: consideration of water density as f(t) → basis: uncertainty of temperature measurement

- **temperature changes in the pipes system**: temperature differences cause volume differences

sources of measurement uncertainty – overview - 2

- **air**: different possible influences like dissolved air is interpreted as a measurment volume, air is compressible
- **diverter**: optimation of diverting operation → redidual error
- **humidity measurement** → uncertainty of humidity sensors and calibration uncertainty
- **master meter**: influence of temperatur, short term stability, calibration uncertainty
- **flow fluctuations**: nonlinear influence on the error curve
- **Start-Stop-mode**: switch on/off → principal systematic influence

model equation

indicated volume | indication meter | balance | density bouyancy | cooling of pipes | air | diverter

$$V_N = \frac{m_N}{\rho} + \delta V_A + \delta V_w + \delta V_{D,A} + \delta V_{Abk} + \delta V_L + \delta V_U + \\ + \delta V_{Master} + \delta V_F + \delta V_{dyn} + \delta V_{SS} + \ldots$$

master meter | influence of humidity | **Q**-fluctuations | start-stop mode

resolution of meter under test and master meter

indication of mut and master meter: V_N $(V_M) \rightarrow$
resolution of volume δV_N:

$$\left(V_N \pm \frac{\delta V_N}{2}\right)$$

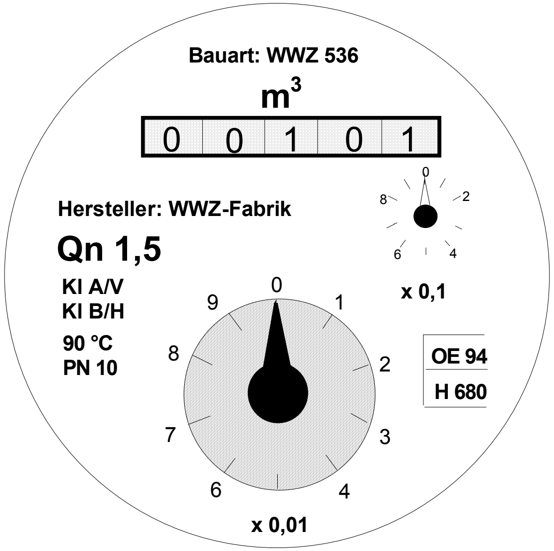
δV_P ... scale interval or pulse: $a_p = \delta V_N / V_N$

variance contribution:

$$u_A^2 = \frac{1}{3}\left(\frac{\delta V_N}{2\,V_N}\right)^2 = \frac{\delta V_N^2}{12\,V_N^2}$$

reed switch: uncertainty ≈ 1/10 od scale interval: $\delta V_{RK}/10$:

$$u_{A,RK}^2 = \frac{1}{3}\left(\frac{\delta V_{RK}}{20\,V_N}\right)^2 = \frac{\delta V_{RK}^2}{1200\,V_N^2}$$



Principally: measurement deviations are of systematic nature
error curves of balances are approximated by a polynome

Important for mu:

- **repeatibility of calibrations**

Waage 1	u_W	u^{*2}_W	Waage 2	u_W	u^{*2}_W	Waage 3	u_W	u^{*2}_W
Last in kg	in g	-	Last in kg	in g	-	Last in kg	in g	-
10	0,1	$1{,}76.10^{-10}$	100	1,6	$4{,}49.10^{-10}$	1000	28,9	$1{,}46.10^{-9}$
30	0,5	$4{,}88.10^{-10}$	200	2,7	$3{,}20.10^{-10}$	2000	57,7	$1{,}46.10^{-9}$
50	0,5	$1{,}76.10^{-10}$	500	8	$4{,}49.10^{-10}$	2900	57,5	$6{,}9.10^{-10}$
100	0,1	$1{,}76.10^{-12}$						

- **long term stability**: if the measurement value x changes by Δx during the time Δt, then one yields for the expectation value and the variance:

$$\mu = \int_0^{t_1} k\, x\, x\, dx = \ldots = \frac{2}{3}\, x_1$$

$$\sigma^2 = \int_0^{t_1} k\, x\, x^2\, dx - \frac{4}{9}\, x_1^2 = \ldots = \frac{1}{18}\, x_1^2$$

use Wagenbreth-Blanke-formula for $\rho(t)$

Contributions to mu

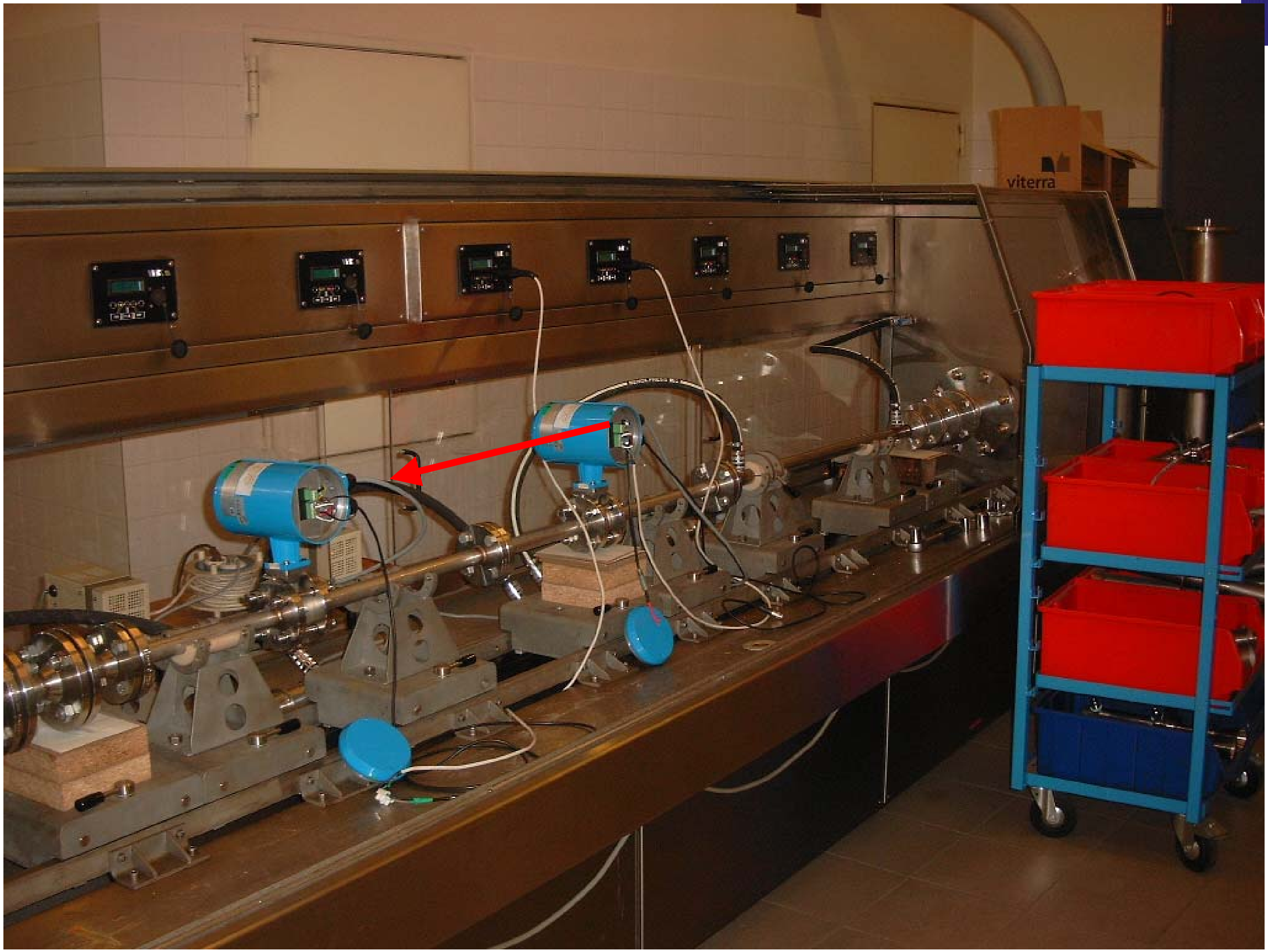
- **influence of temperature measurement on density ➔**

$$u^2_{D,A1} = \left(\frac{\partial V(t)}{\partial t}\right)^2 u_t^2$$

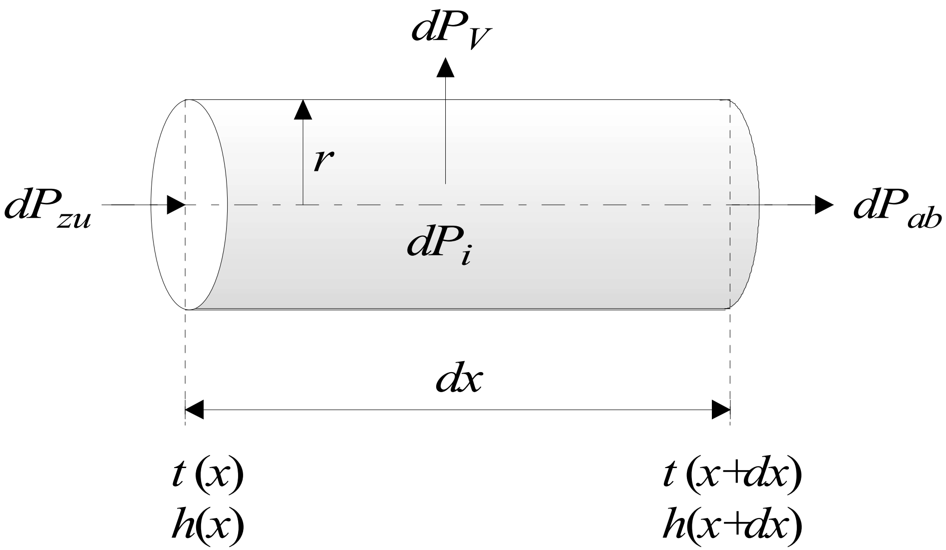
- **actual value of density at defined temperature** $\delta\rho \approx 5.10^{-5}$ ➔

$$u^2_{DA,2} = 8{,}3.10^{-10}$$

view of test rig ... cooling problem



cooling of the pipe system - 1



cooling of pipe system depending on

- flow rate: ***dV/d*** τ
- distance from inlet: ***x***
- heat transfer coefficient ***k***
- density ρ and specific heat c_p of heat carrier

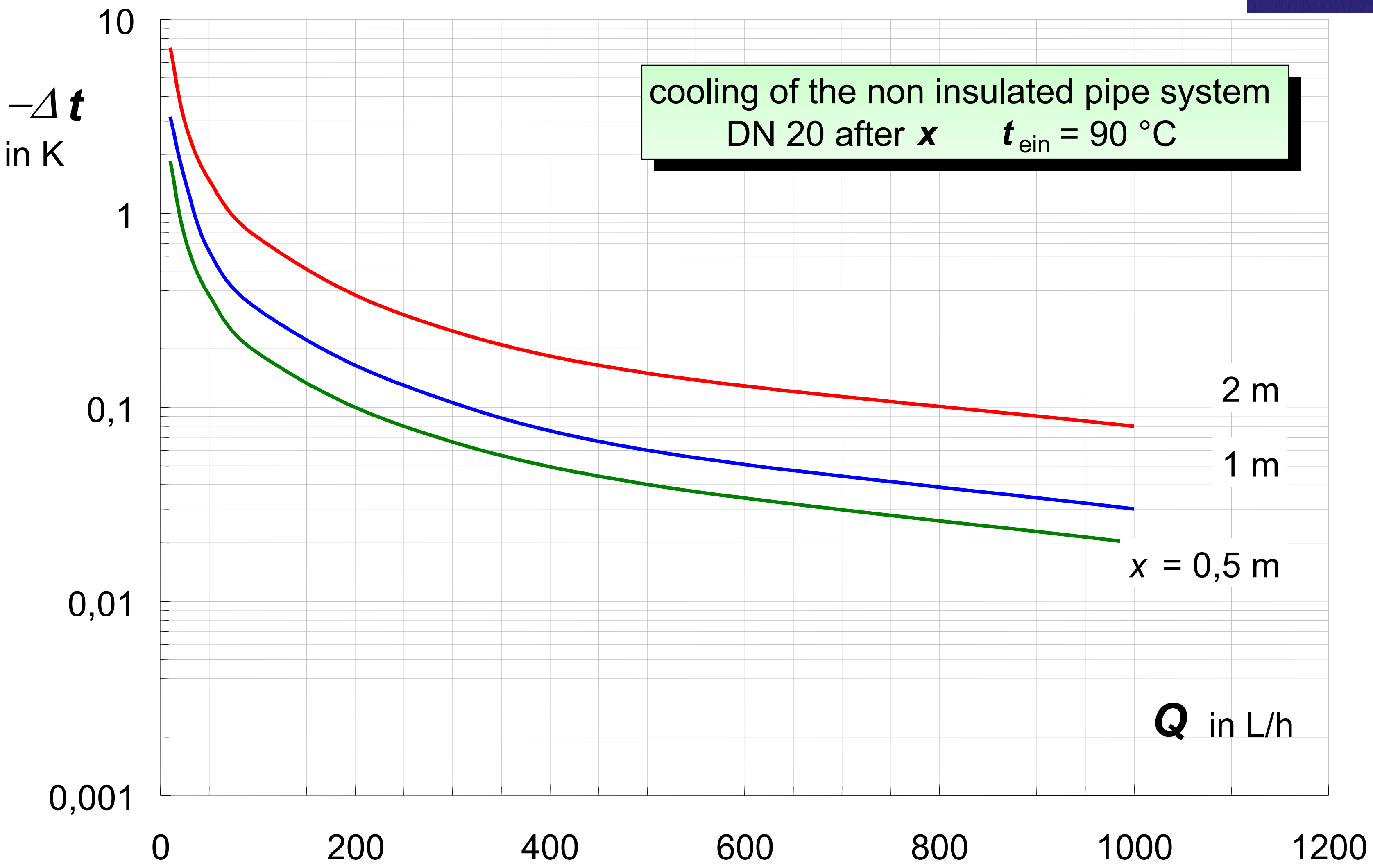
temperature difference heat carrier and sourrounding: $\Theta(x)$

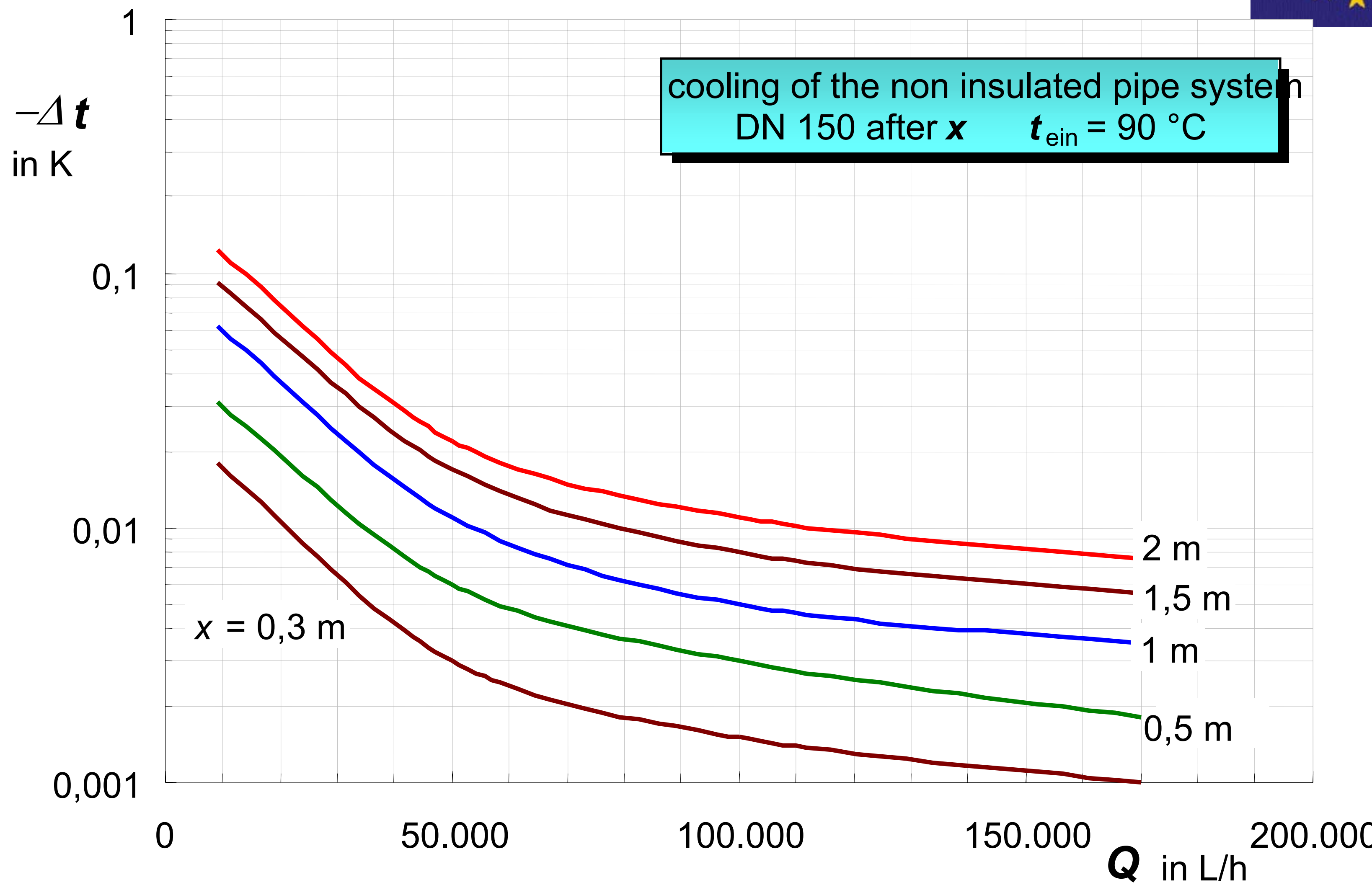
maximal temperature difference at the inlet of test rig: $\Theta_e(x)$

result:

$$\frac{\Theta}{\Theta_e} = e^{-\frac{C_1}{\dot{V}}x} \quad \text{mit} \quad C_1 = \frac{2\pi r k}{c_p \rho}$$

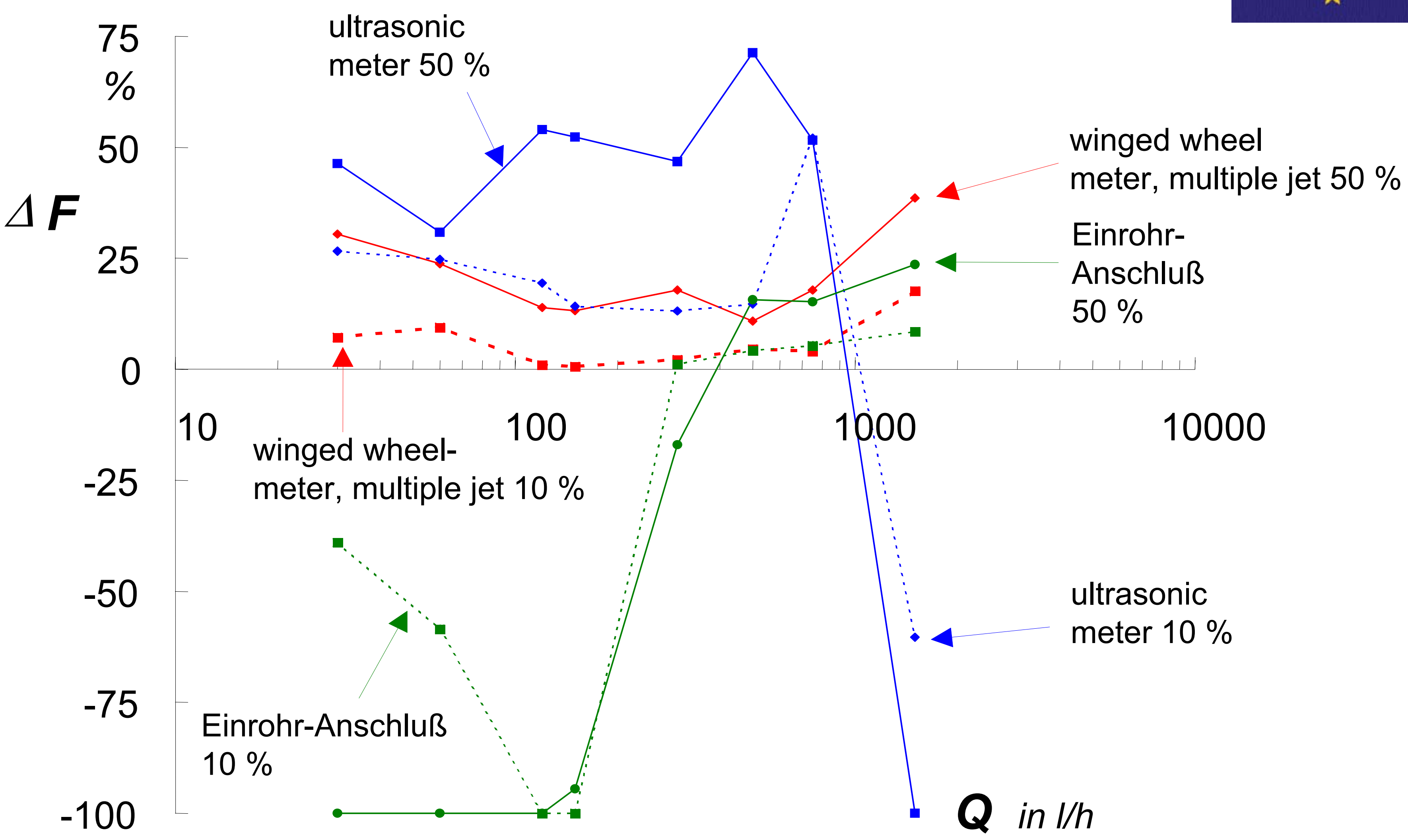
cooling of the pipe system – DN 20





cooling of the non insulated pipe system
DN 150 after *x* t_{ein} = 90 °C
$-\Delta t$ in K
1
0,1
0,01
0,001
0
50.000
100.000
150.000
200.000
Q in L/h
2 m
1,5 m
1 m
0,5 m
x = 0,3 m

air in the pipe system – experiences



air in the pipe system - variances

In principle: air proportional V_L is transported ➔

$$u_L^2 = \frac{1}{3}\left(\frac{V_L}{V_N}\right)^2$$

actual: air in dead corners etc. ➔ air as gas is compressed (e.g. start-stop-mode):

$$u_{L1}^2 = \frac{1}{3}\frac{\left[V_L(1-\frac{p_1}{p_2})\right]^2}{V_N^2}$$

If air undergoes temperatur changes Δt ➔ Boyle-Mariotte:

$$\Delta V_L = \frac{1}{273}\Delta t V_L$$

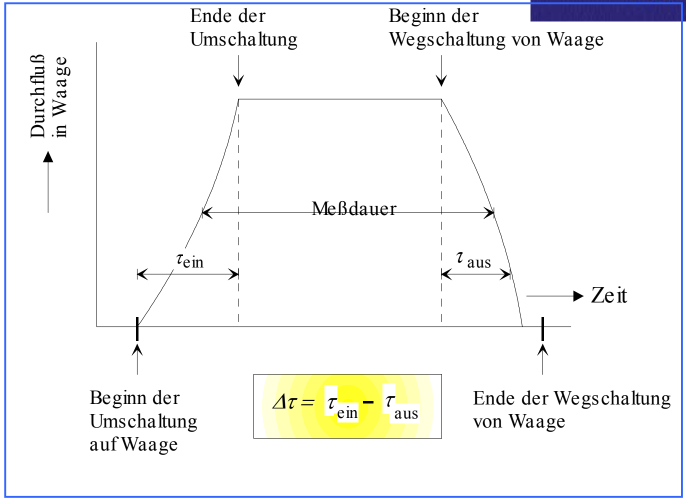
➔

$$u_{L2}^2 = \frac{1}{3}1{,}34.10^{-5}\Delta t^2\left(\frac{V_L}{V_N}\right)^2$$

influence of not registered volumen

ΔV_u ➔

$$u_u^2 = \frac{1}{3}\frac{\Delta V_u^2}{V_N^2}$$



measurement values

Kleiner Diverter		*Großer Diverter*	
Q in L/h	*Δτ* in ms	***Q*** in L/h	*Δτ* in ms
120	30,4	7500	10,7
240	-14,5	10000	11,8
500	-4,5	15000	11,6
1000	-2	20000	3,3
1500	-1,5	50000	1,7
2000	-1,9	100000	0,6
3000	-3,2	150000	-6,4

big diverter

$u^2(f)$ is the experimentally determined measurement uncertainty of the humidity sensor ➔

$$\frac{u^2(W_P)}{W_P^2} \approx 2.10^{-6} \left[2 \left(\frac{V_B}{V_P} \right)^2 + 1 \right] u^2(f)$$

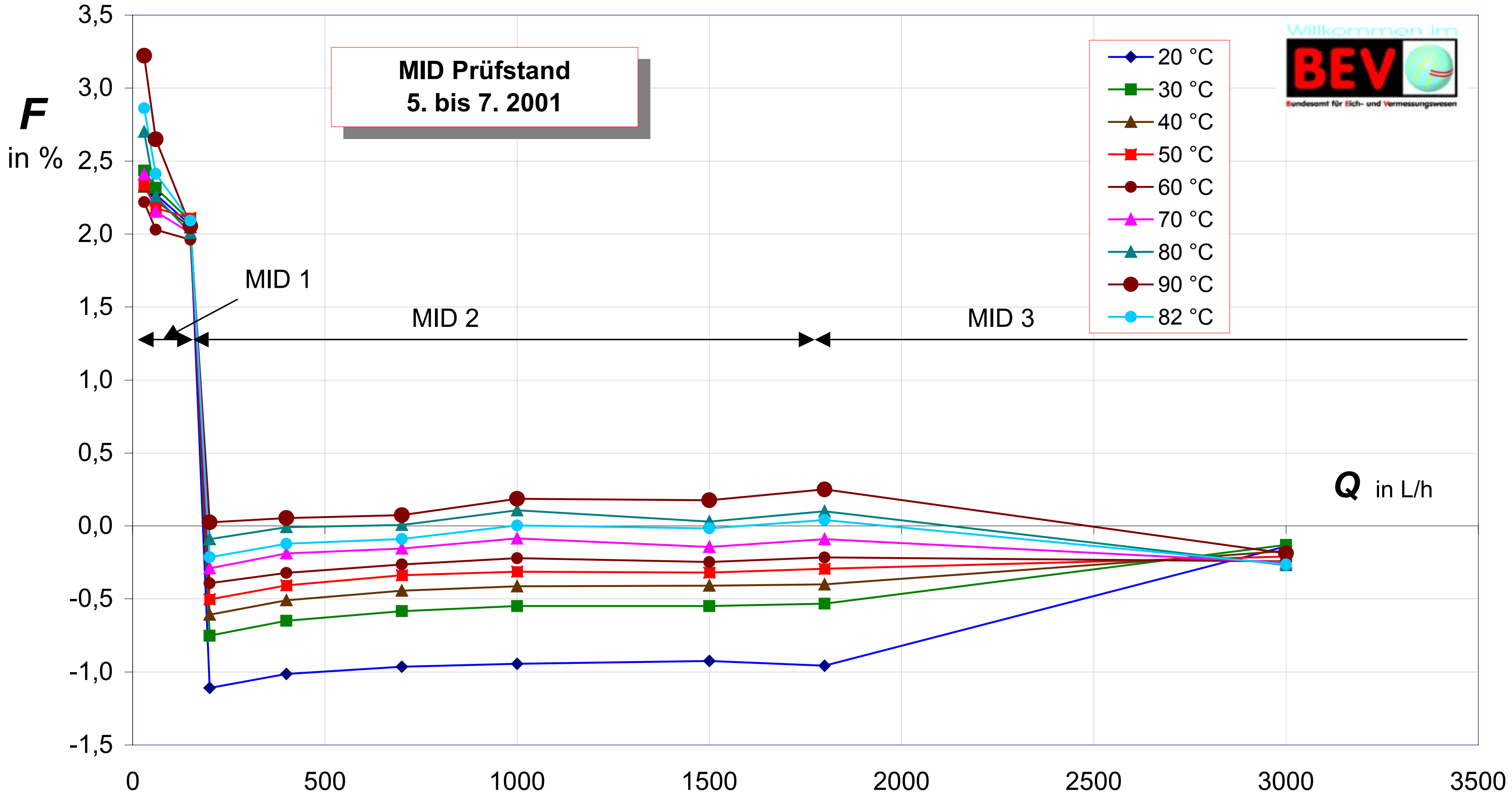
$\boldsymbol{W_P}$ is the conventional value

Result: The influence of the variance contribution of the humidity measurements depends on

- the measurement uncertainty of tghe humidity sensor and
- of the filling degree of the container: container volumen V_B /test volume V_P

Mastermeter

- important: **short term stability**
- very small **temperature dependency**



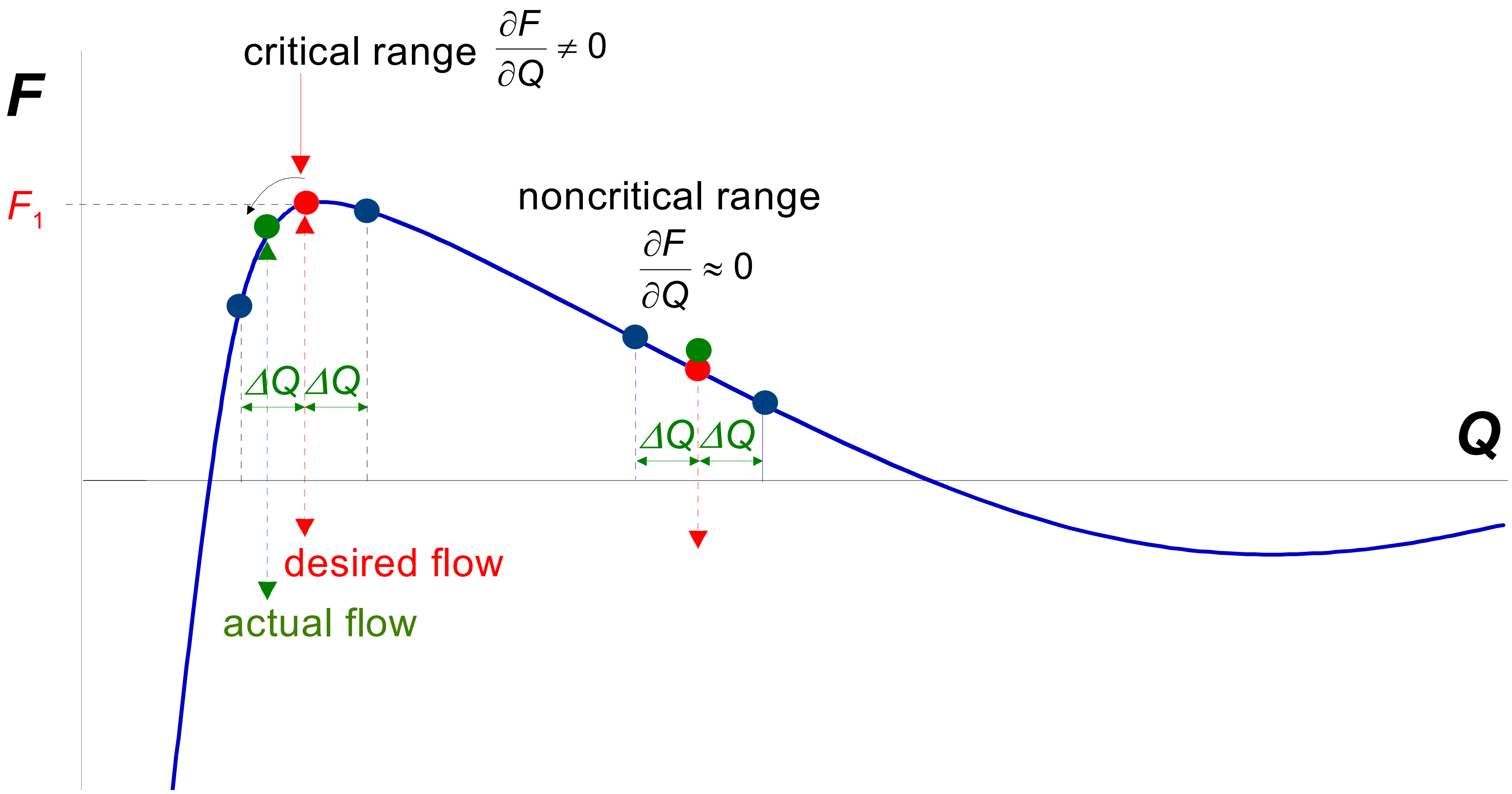
Important is:

$$\boxed{\varepsilon = \frac{\partial F}{\partial Q}}$$

if the error curve linear $\Rightarrow$ influence of fluctuations vanishes

If $(\partial F/\partial Q) > 0 \Rightarrow$ influence is not neglectible

Procedure: first find analytic value for $\boldsymbol{F}(\boldsymbol{Q})$
➔ then $\boldsymbol{u}^2(\Delta \boldsymbol{Q})$ ➔ see example


critical range $\frac{\partial F}{\partial Q} \neq 0$
F
F_1
noncritical range
$\frac{\partial F}{\partial Q} \approx 0$
ΔQ ΔQ
ΔQ ΔQ
Q
desired flow
actual flow

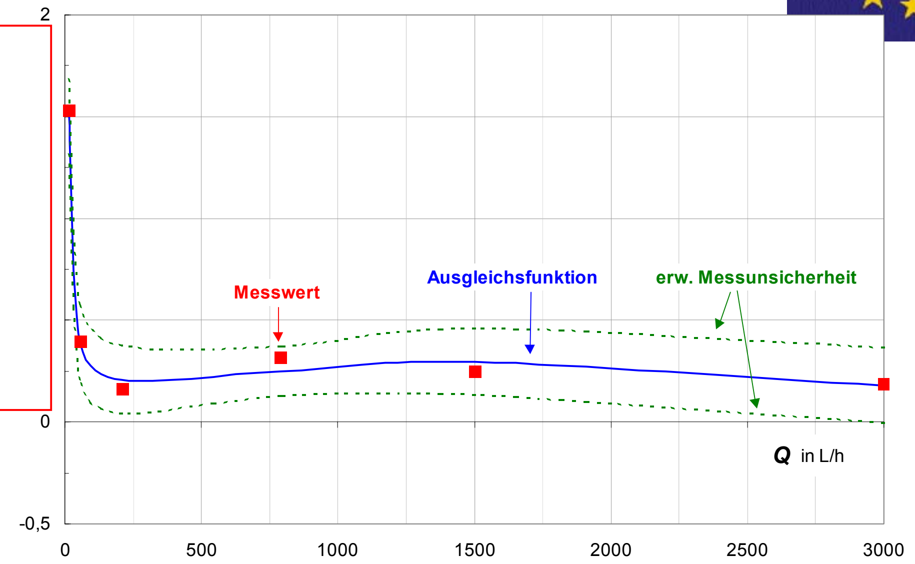
Procedure

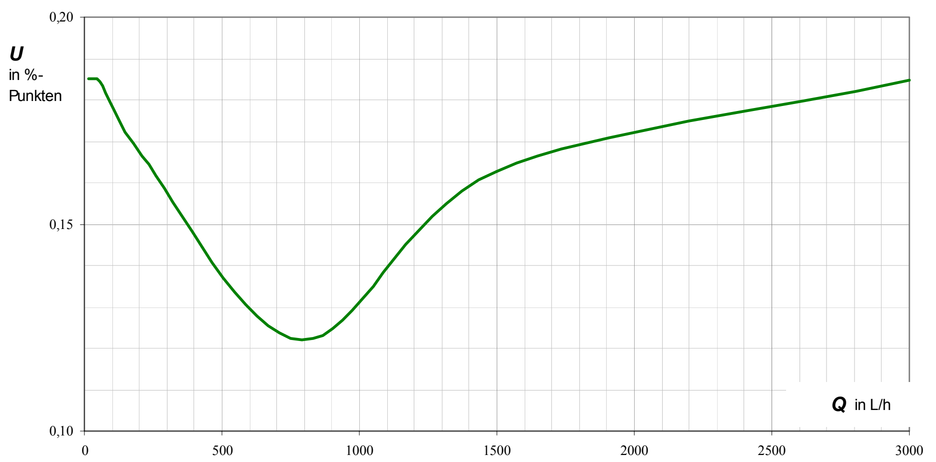
- assumption of the theoretical error curve shape $F(Q)$
- calculate the coefficients
- mu of coefficients
- expandes uncertainty

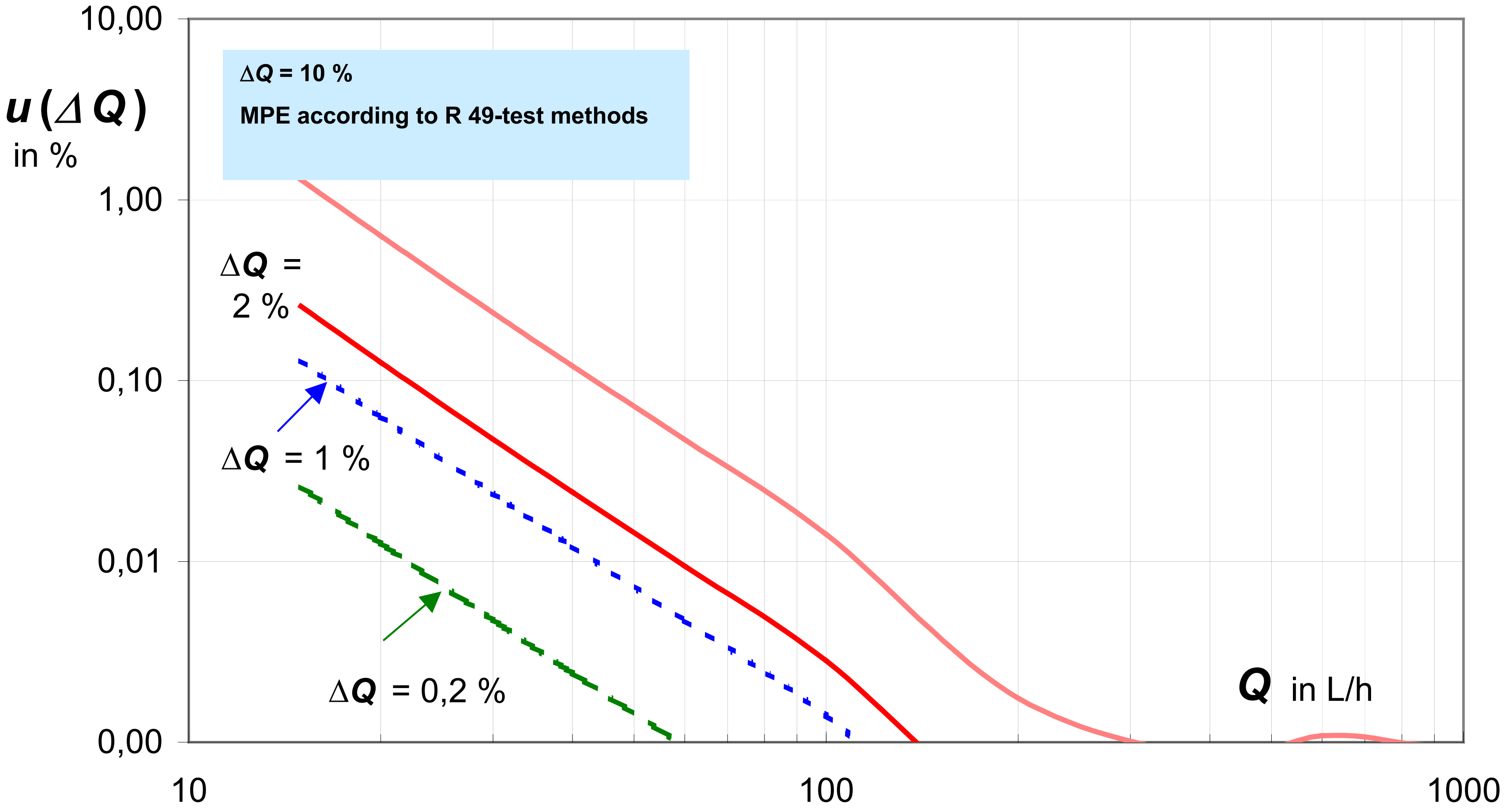
$$F(Q) = a_0 + a_1 Q + a_2 Q^2 + a_3 Q^{-1} + a_4 Q^{-2}$$

$a_0 =$	$9{,}32.10^{-2}$
$a_1 =$	$2{,}295.10^{-4}$
$a_2 =$	$-6{,}741.10^{-8}$
$a_3 =$	$14{,}00$
$a_4 =$	$112{,}9$

$$u_c^2 = \sum_{i=0}^{4} \left(\frac{\partial F}{\partial a_i} \right)^2 u^2(a_i)$$






10,00
u(ΔQ)
in %
1,00
0,10
0,01
0,00
10
100
1000
ΔQ = 10 %
MPE according to R 49-test methods
ΔQ = 2 %
ΔQ = 1 %
ΔQ = 0,2 %
Q in L/h

Start-stop-mode - 1

applied to meters without output pulses ➔ most domestic meters

Result of a winged wheel type water meter

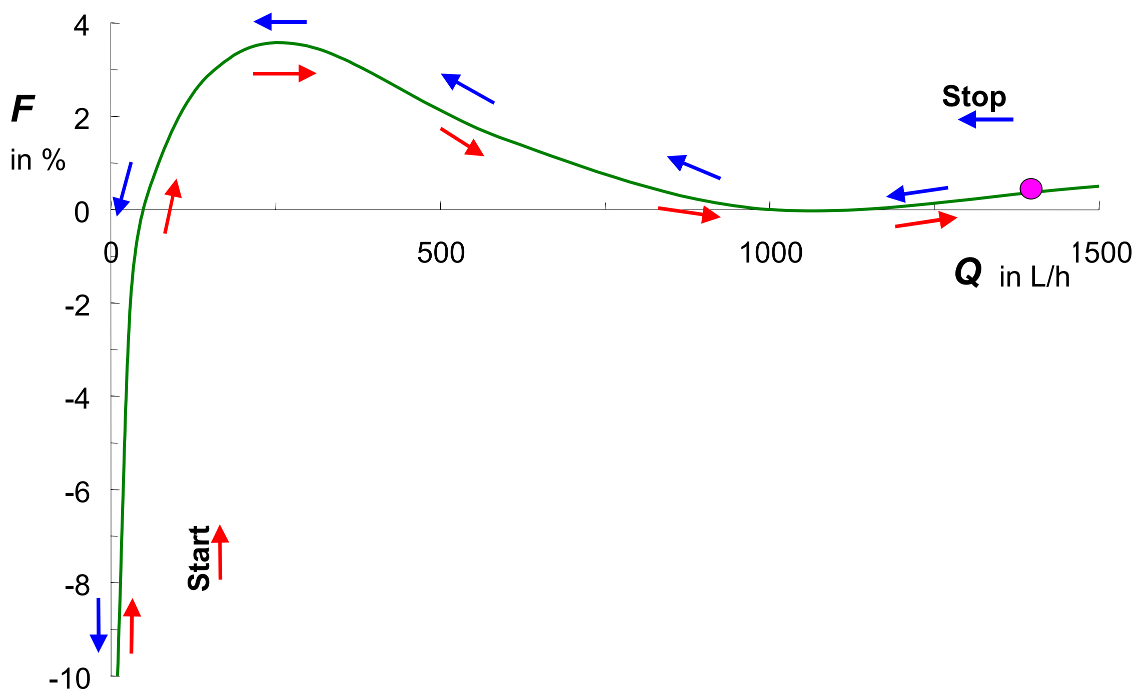
Q [L/h]	V [L]	$x_{0,ss} - x_{0,f}$ [%]	$\Delta u_{ss,f}$
1500	50	0,13	0,033
1500	100	0,16	0,028
1500	200	0,09	0,006
1500	500	0,08	0,002
1500	1000	0,08	0,005

$X_{0,f}$ ... indication at dynamic mode (mean of several measurements)

$X_{0,ss}$ ... indication of start-stop-mode (mean of several measurements)

$\Delta u_{ss,f}$ ... variance contribution

Systematic effect: At start-stop-mode the error curve is passed twice with a different velocity ➔ calculation of mean over time



typical systematic effect: - 0,1 % (measured) ➔ variance contribution

EMATEM

Fliegender Betrieb mit Waage; Zählerimpulse synchronisiert

Grüne Bereiche: Werte eingeben

Q_n in m³/h	**150**
Wassertemperatur [°C]	**40**
Umgebungstemperatur [°C]	**25**
Luftdruck [mbar]:	**1.000**

Durchfluss [l/h]	170000	150000	100000	50000	25000	15000	9000
Prüfvolumen V_P [L]	2800	2800	2800	2800	2800	2800	2800
ausgewählte Waage	3000	3000	3000	3000	3000	3000	3000
Prüfzeit [s]	59	67	101	202	403	672	1120
Ausgangsimpulse des Prüflings [Impulse/L]	1,0	1,0	1,0	1,0	1,0	1,0	1,0
Ausgangsimpulse des Masterzählers [Impulse/L]	10	10	10	10	10	100	100
Wiederholbarkeit des Prüflings $u(x_i)$ [%]	0,011	0,029	0,033	0,022	0,033	0,045	0,045
Rohrvolumen zwischen Prüfling und Masterzähler [L]	56	56	56	56	56	56	56
Luftanteil im Wasser [L]	0,3	0,3	0,3	0,3	0,3	0,3	0,3
Abkühlung der Luftblase [K]	5	5	5	5	5	5	5
Messunsicherheit der Temperatur für die Dichte- und Auftriebskorrektur [K]	0,10	0,10	0,10	0,10	0,10	0,10	0,10
Temperaturunterschied zwischen Prüfling und Masterzähler [°C]	0,10	0,10	0,11	0,14	0,16	0,18	0,21
Zahl der Wiederholmessungen	10	10	10	10	10	10	10
absolute Luftfeuchte [g/m³] bei der Temperatur des Wassers	51,1	51,1	51,1	51,1	51,1	51,1	51,1
absolute Luftfeuchte [g/m³] bei Umgebungstemperatur	23,1	23,1	23,1	23,1	23,1	23,1	23,1
Verdampfungs-Luftstrom [g]	78	78	78	78	78	78	78
Umschaltzeit des Diverters [ms]	-6,4	-6,4	0,6	1,7	3,3	11,6	11,8

Berechnung der Erw. Messunsicherheit – Ergebnisse

Auflösung des Prüflings: u^2_A	0,00	0,00	0,00	0,00	0,00	0,00	0,00
Auflösung des Masterzählers: u^2_{MZ}	4,25E-10	4,25E-10	4,25E-10	4,25E-10	4,25E-10	4,25E-12	4,25E-12
Waage u_w^2	8,40E-10	8,40E-10	8,40E-10	8,40E-10	8,40E-10	8,40E-10	8,40E-10
Langzeitstabilität der Waage: $u_{w,LT}^2$	5,00E-09	5,00E-09	5,00E-09	5,00E-09	5,00E-09	5,00E-09	5,00E-09
Dichte und Auftriebskorrektur 1: temperature $u_{D,A1}^2$	1,51E-09	1,51E-09	1,51E-09	1,51E-09	1,51E-09	1,51E-09	1,51E-09
Dichte und Auftriebskorrektur 2: density $u_{D,A2}^2$	4,10E-10	4,10E-10	4,10E-10	4,10E-10	4,10E-10	4,10E-10	4,10E-10
Temperaturabfall zwischen Prüfling und Masterzähler: $u_{TÄ}^2$	3,23E-13	3,44E-13	4,21E-13	5,95E-13	8,41E-13	1,09E-12	1,40E-12
Gelöste Luft im Rohr (1): u_{L1}^2 (nur für Start-Stopp-Betrieb)	0E+00	0E+00	0E+00	0E+00	0E+00	0E+00	0E+00
Gelöste Luft im Rohr (2): u_{L2}^2	1,28E-12	1,28E-12	1,28E-12	1,28E-12	1,28E-12	1,28E-12	1,28E-12
Diverter: u_u^2	3,88E-09	3,02E-09	1,18E-11	2,37E-11	2,23E-11	9,93E-11	3,70E-11
Feuchtigkeitseffekte: u_f^2	1,68E-11	8,42E-10	8,42E-10	8,42E-10	8,42E-10	8,42E-10	8,42E-10
Wiederholbarkeit des Prüflings: $u^2(x_i)$	1,21E-08	8,21E-08	1,09E-07	4,64E-08	1,06E-07	2,02E-07	2,00E-07
Kombinierte Varianz: u_c^2	2,42E-08	9,41E-08	1,18E-07	5,54E-08	1,15E-07	2,11E-07	2,08E-07
Erweiterte Messunsicherheit *U*: [%] für *k* = 2	*0,031*	*0,061*	*0,069*	*0,047*	*0,068*	*0,092*	*0,091*
ν_{eff}	19,09	11,64	10,48	12,36	10,52	9,77	9,78
k_{eff}	2,11	2,24	2,28	2,21	2,28	2,31	2,31
***Erweiterte Messunsicherheit: U_{eff}* [%]**	***0,033***	***0,069***	***0,078***	***0,052***	***0,077***	***0,106***	***0,106***
Kombinierte Varianz: u_c^2 ohne Wiederholbarkeit des Prüflings	1,21E-08	1,21E-08	9,04E-09	9,05E-09	9,05E-09	8,71E-09	8,64E-09
Erweiterte Messunsicherheit *U*: [%] für *k* = 2	***0,022***	***0,022***	***0,019***	***0,019***	***0,019***	***0,019***	***0,019***
ν_{eff}	10,16	11,71	8,74	8,76	8,76	8,16	8,04
k_{eff}	2,29	2,23	2,37	2,37	2,37	2,42	2,43
Erweiterte Messunsicherheit *U_{eff}* ohne Wiederholbarkeit des Prüflings	***0,025***	***0,025***	***0,023***	***0,023***	***0,023***	***0,023***	***0,023***