Humidity effects of flow measurements with open containers

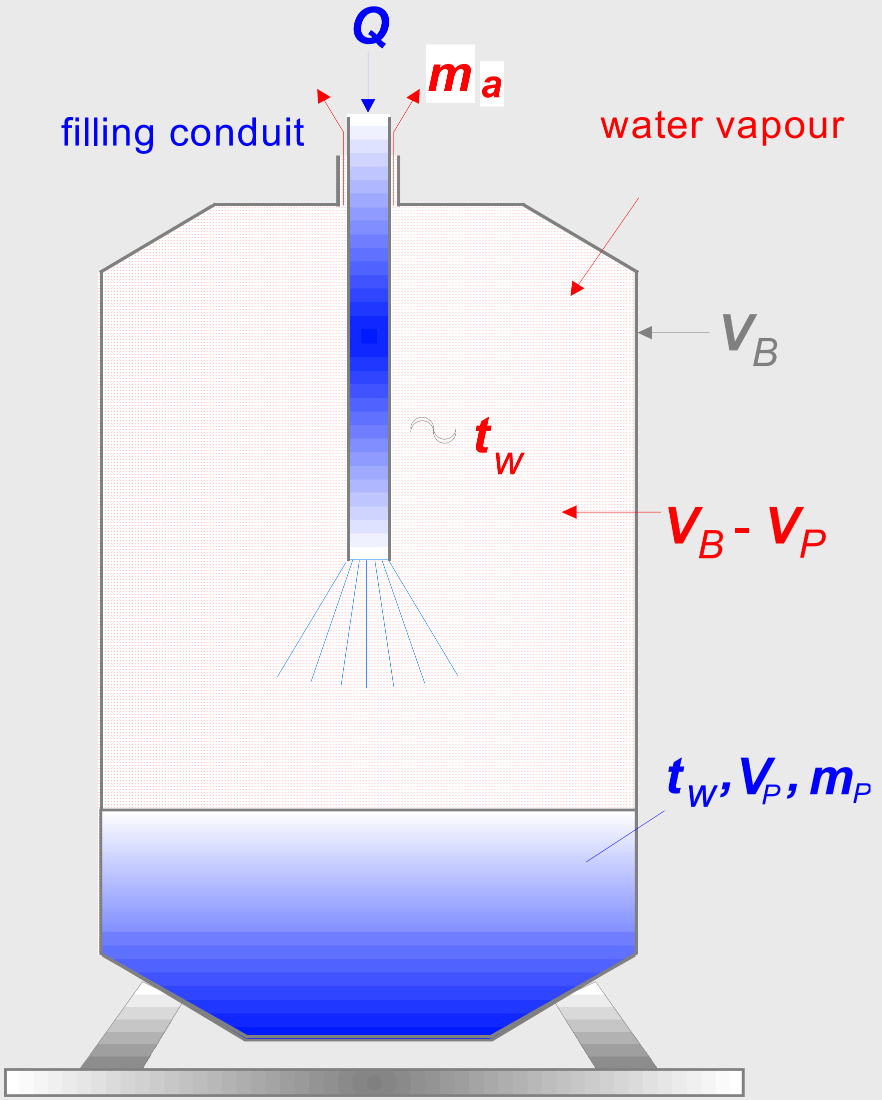
Situation

A container on a balance as a part of a water flow test rig is a nearly closed container with a little aperture for the filling conduit. The filling conduit is not in contact with the container

The separated air can leave the container at the left cross section

Determination of the filled water in the container is done by weighing before and after the filling and by difference calculation

Container with water (blue) and vapour (red dotted)



$$W_1\left(1-\frac{\rho_{Lu}}{\rho_N}\right) = m_B + m_{w1} + m_{L1} - A \qquad (1)$$

W_1 ... result of weighing before filling. W_1 is the conventional mass
➔ less than the mass, if the density
is less than density of reference (8 000 kg/m³)

m_B ... mass of the container

m_{W1} ... mass of the water in the container

V_B ... volume of the container

m_{L1} ... mass of air in the container: $m_{L1} = \left(V_B - \frac{m_{W1}}{\rho_{W1}}\right)\rho_{L1}$ (2)

A ... buoyancy = $V_B\,\rho_{Lu}$ (3)

ρ_{L1} ... density of air in the container before filling

ρ_{L2} ... density of air in the container after filling

$\rho_{Lu,1}$ ... density of air outside the container

Second weighing procedure (after filling procedure)

$$W_2\left(1-\frac{\rho_{Lu}}{\rho_N}\right)=m_B+m_{w2}+m_{L2}-A \qquad (4)$$

W_2 ... result of weighing after filling

m_{W2} ... mass of the water in the container

m_{L2} ... mass of air in the container: $m_{L2}=\left(V_B-\frac{m_{W2}}{\rho_{W2}}\right)\rho_{L2}$ (5)

$$(W_2-W_1)\left(1-\frac{\rho_{Lu}}{\rho_N}\right)=$$

$$=m_{W2}+\left(V_B-\frac{m_{W2}}{\rho_{W2}}\right)\rho_{L2}-\cancel{V_B\rho_{Lu,2}}-m_{W1}-\left(V_B-\frac{m_{W1}}{\rho_{W1}}\right)\rho_{L1}+\cancel{V_B\rho_{Lu,1}} \qquad (6)$$

The density of air outside the container is nearly equal before and after filling. Therefore, the corresponding terms in equ. (6): ($V_B\,\rho_{Lu,1}$ and $V_B\,\rho_{Lu,2}$) disappear.

Density of air

The density of air in the container is different at the first and the second reading, because temperature and humidity are different. We get:

$$
\begin{aligned}
(W_2 - W_1)(1 - \frac{\rho_{Lu}}{\rho_N}) &= m_{W2} + \left(V_B - \frac{m_{W2}}{\rho_{W2}}\right)\rho_{L2} - m_{W1} - \left(V_B - \frac{m_{W1}}{\rho_{W1}}\right)\rho_{L1} = \\
&= m_{W2}\left(1 - \frac{\rho_{L2}}{\rho_{W2}}\right) - m_{W1}\left(1 - \frac{\rho_{L1}}{\rho_{W1}}\right) + V_B(\rho_{L2} - \rho_{L1})
\end{aligned}
\qquad (7)
$$

Special case: filling in the empty container

$m_{w1} = 0$

From equ. (7) we get

$$\Delta W\ (1 - \frac{\rho_{Lu}}{\rho_N}) = m_{W2}\left(1 - \frac{\rho_{L2}}{\rho_{W2}}\right) + V_B(\rho_{L2} - \rho_{L1}) \quad \text{resp.} \tag{8a}$$

and

$$\boxed{m_{W2} = \frac{\Delta W\ (1 - \frac{\rho_{Lu}}{\rho_N}) - V_B(\rho_{L2} - \rho_{L1})}{\left(1 - \frac{\rho_{L2}}{\rho_{W2}}\right)}} \tag{8b}$$

Consideration of absolute humidity of air

Definitions:

$V_B\, f_1$ ... absolute humidity before filling in the empty container [m³xkg/m³]

$V_{L2}\, f_2$ ... absolute humidity after filling procedure

m_P ... mass of water filled in the container

m_a ... absolute humidity of the effluent air flow

With this definitions we get: $$V_{L2}\, f_2 = \left(V_B - \frac{m_{w2}}{\rho_{W2}} \right) f_2 \qquad (9)$$

Bilance of the masses: $$m_{w2} + V_{L2}\, f_2 = V_B\, f_1 + m_P - m_a$$

For the mass of water after filling we get:

$$\boxed{m_P = m_{w_2}\left(1 - \frac{f_2}{\rho_{W2}}\right) + V_B(f_2 - f_1) + m_a} \qquad (10)$$

Result - 1

m_{W2} from equ. (8b) in equ. (10) gives equ. (11):

$$m_P = \frac{(1 - \frac{f_2}{\rho_{w2}})\left[\Delta W\,(1 - \frac{\rho_{Lu}}{\rho_N}) - V_B\,(\rho_{L2} - \rho_{L1})\right]}{(1 - \frac{\rho_{L2}}{\rho_{w2}})} + V_B\,(f_2 - f_1) + m_a \qquad (11)$$

For m_a - absolute humidity of the effluent air flow - we get

$$m_a = \int_o^{t_P} f(t)\frac{dV_L}{dt}\,dt = \int_o^{t_P} f(t)\,Q\,dt \qquad (12)$$

What is to do?

(1) measurement of absolute humidity before and after filling (f_1 and f_2)

(2) measurement of temperatures and

(3) the mass flow:

$$m_a = \int_o^{t_P} f(t)\frac{dV_L}{dt}\,dt = \int_o^{t_P} f(t)\,Q\,dt$$

Uncertainty contribution of Δf

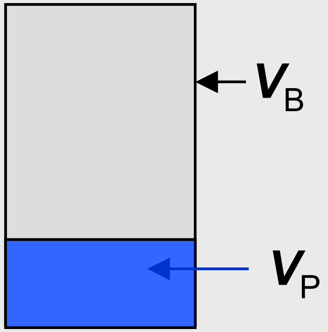
By taking into account the actual air density and the humidity losses according to equation (11), the actual water volume can be determined. In fact, however, two measurement values may be affected by errors. These are the temperature — already considered by density and buoyancy uncertainty – and humidity: $u(\Delta f)$:

The effect of uncertainty is:

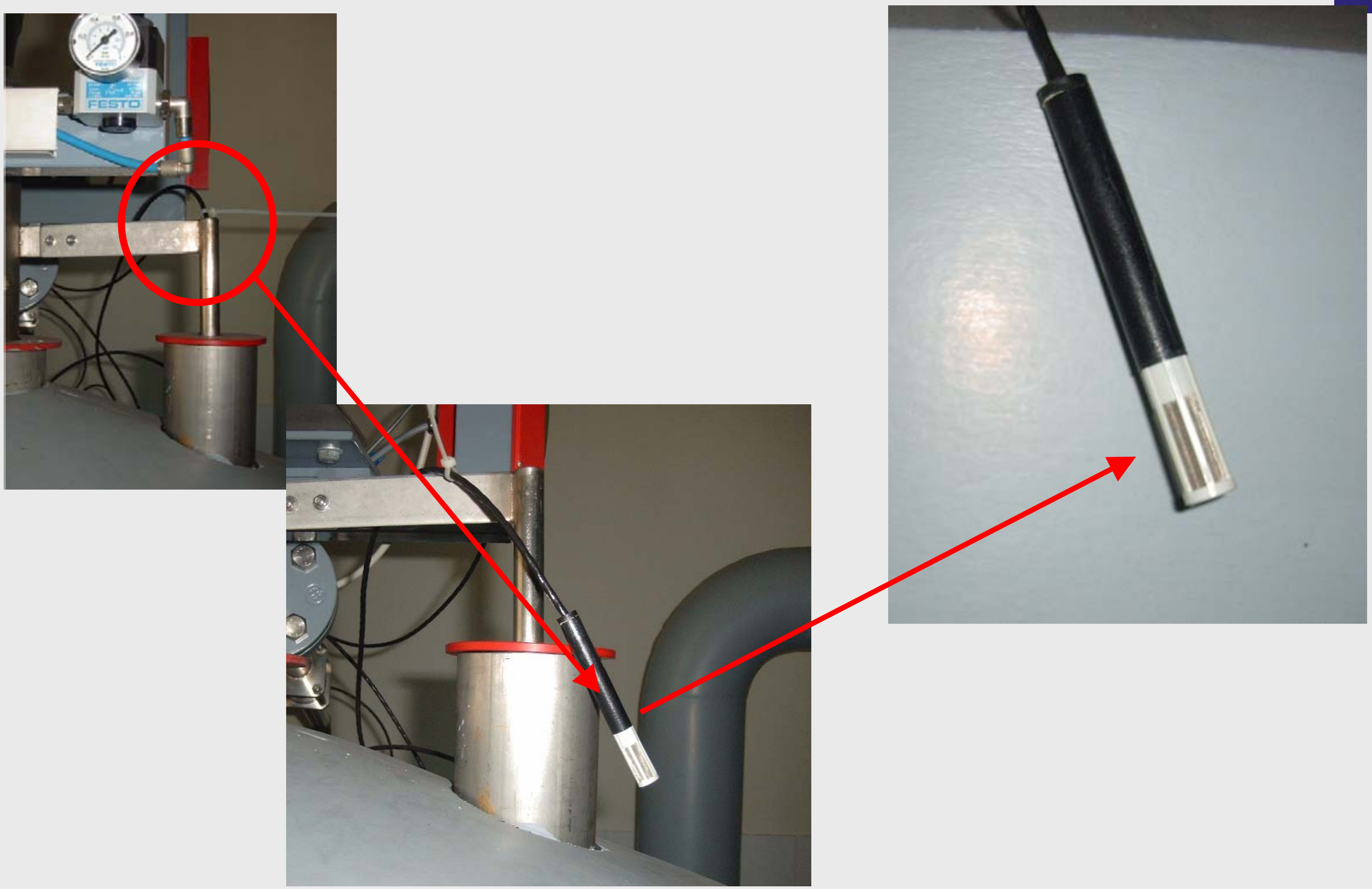
$$\frac{u^2(m_P)}{m_P^2} \approx 2.10^{-6}\left[2\left(\frac{V_B}{V_P}\right)^2 + 1\right] u^2(f)$$

V_P/V_B	$u^2(m_P)/m_P^2$	$u(m_P)/m_P$
1	$1{,}3 \times 10^{-10}$	$1{,}1 \times 10^{-5}$
0,5	$3{,}8 \times 10^{-10}$	$1{,}9 \times 10^{-5}$
0,1	$8{,}4 \times 10^{-9}$	$9{,}2 \times 10^{-5}$
0,01	$8{,}4 \times 10^{-7}$	$9{,}2 \times 10^{-4}$

V_B … Volume of the container
V_P … tested volume



Some pictures of the humidity sensor



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