EMATEM

European Metrology Association
for Thermal Energy Measurement

THERMAL ENERGY METER
(DYNAMIC MODEL)

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Introduction –

An overview on thermal energy meter

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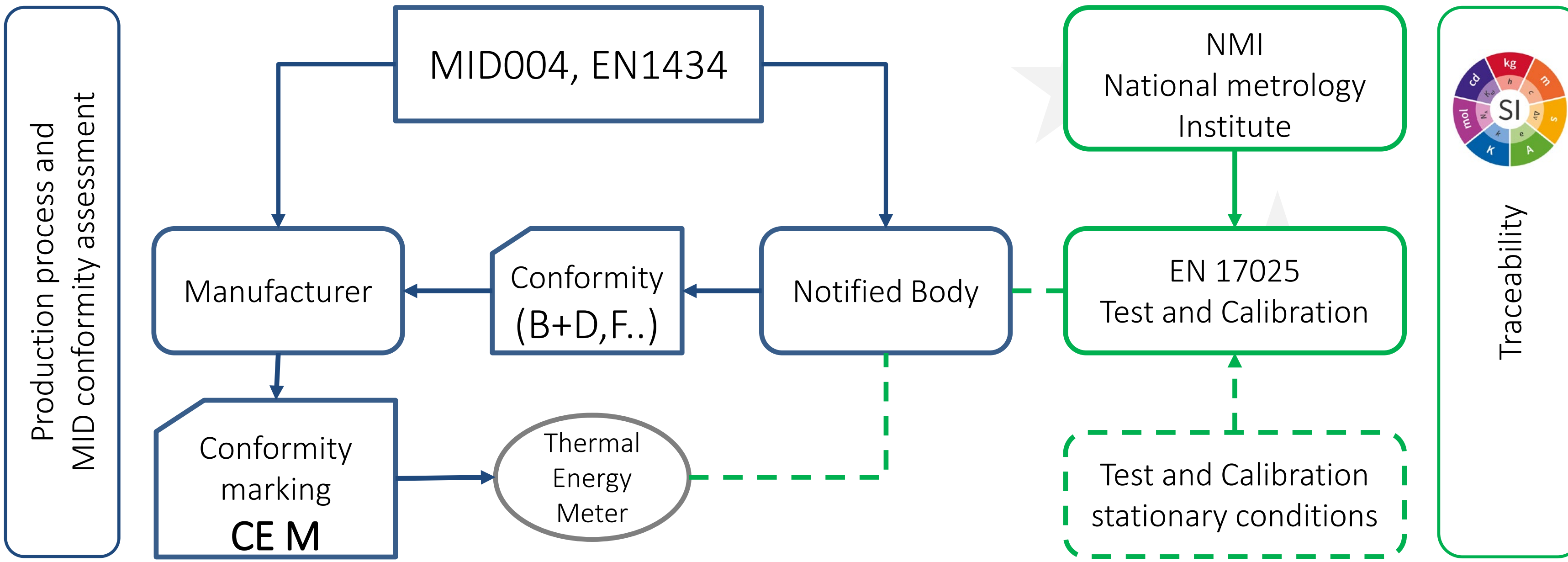
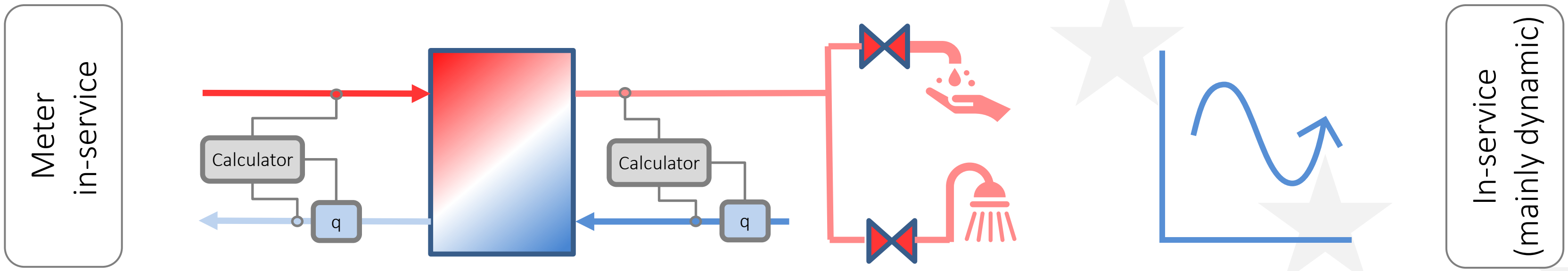


// Thermal energy meter – whole process

Evidence from in service meters of dynamic effects on measurement



- Most of field application are close to be “near stationary“ (negligible dynamic effect).
- In some case errors, due to dynamic effect, may be hidden by meter accuracy ($E \leq E_c + E_q + E_\theta$).
- Evident Dynamic effect on few particular application. (*)

(*) THERMAL ENERGY METERS WITH SHORT INTEGRATION TIMES - BJÖRN FOLKESON DANIEL MÅNSSON, THOMAS FRANZÉN - ISBN 978-91-7673-561-9 | © Energiforsk January 2019 www.energiforsk.se

// Thermal energy meter – whole process

Study points

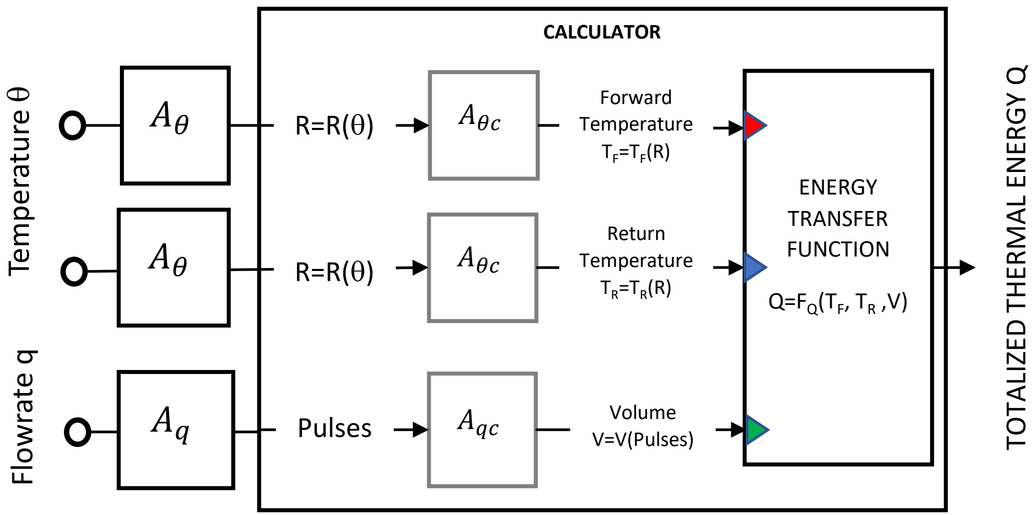
- Define a (simple) dynamic model for thermal energy meters
- Model parameter estimation: test methods and procedures
- Dynamic model and Fast Response
  - ✓ opportunity: *it may be possible and useful to think about a classification of response?*
  - ✓ Improvement: *it would possible knowing HOW FAST is the meter and not only if it is "fast" or "not fast"?*

02

Thermal Energy Meter – An overview on stationary model

// Thermal energy meter – Stationary model

Heat transmission formula



$$A(\theta) \rightarrow R(\theta) = R_0(1 + A\theta + B\theta^2)$$

$$A(q) \rightarrow q = K_{factor} f_{pulses}$$

EN 1434-1:2020

$$Q = \int_{\Delta V} k(\theta_i - \theta_o)dV$$

where:

Q Energy (J)

k heat coefficient (J/dm³/K)

θ_i temperature on forward side (intlet) (°C)

θ_o temperature on return side (outlet) (°C)

V volume of heat conveying fluid (dm³)

03

Thermal Energy Meter –

Proposal of a simple Dynamic model

// Thermal energy meter – Dynamic model

Differenzial equation formula (1st order, linear)

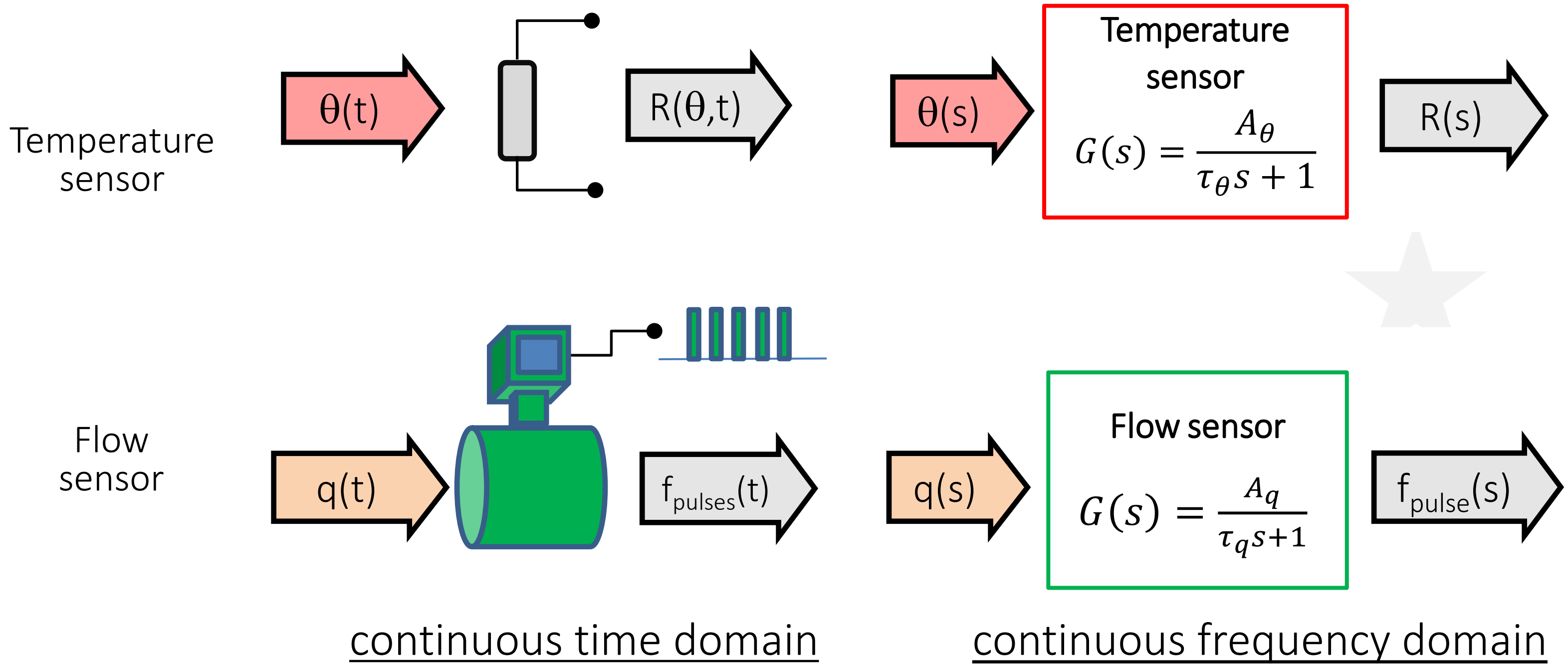
$$a_1 \frac{dy(x)}{dt} + a_0 y(x) = x(t) \Rightarrow L(s) \Rightarrow a_1 sY(s) + a_0 Y(s) = X(s)$$

Where $y(x)$ is the output and $x(t)$ is the input in <u>continuous time domain</u>.
Laplace transform differential equation into algebraic function of the complex variable $s = j\omega$ in <u>continuous frequency domain</u>.

$$G(s) = \frac{Y(s)}{X(s)} = \frac{1}{\tau s + 1} A$$

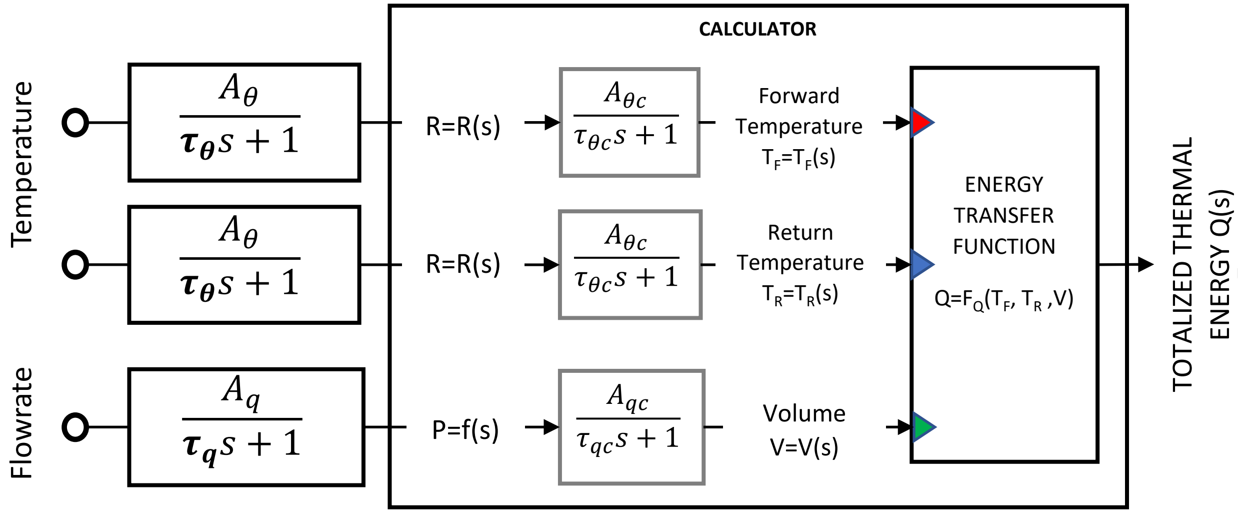
Transfer function **G(s)** is the dynamic model equation where $(\boldsymbol{A}, \boldsymbol{\tau})$ are the <u>only two parameters</u>.
$A(s)$: Stationary response
$\frac{1}{\tau s+1}$: complex part that involves dynamic characterization of response.

// Thermal energy meter – Dynamic model



Note: In the same way is possible to have the complete calculator model in frequency domain.

// Thermal energy meter – Dynamic model



Dynamic model is characterized by four time constants $(\tau_\theta, \tau_{\theta c}, \tau_q, \tau_{qc})$.

If $\tau_\theta \gg \tau_{\theta c}$ and $\tau_q \gg \tau_{qc}$ is possible to reduce parameters to **only two** time constants: $(\boldsymbol{\tau_\theta}, \boldsymbol{\tau_q})$

04

Thermal Energy Meter –

Dynamic model parameter estimation

// Thermal energy meter – Parameter estimation

Step response

$$\begin{cases} X(s) = \dfrac{\Delta_0}{s} \\ G(s) = \dfrac{Y(s)}{X(s)} = \dfrac{1}{\tau s + 1} A \end{cases} \Rightarrow \quad Y(s) = G(s)X(s)$$

Where $G(s)$ is the transfer function and $\mathrm{X}(s)$ is the step input.

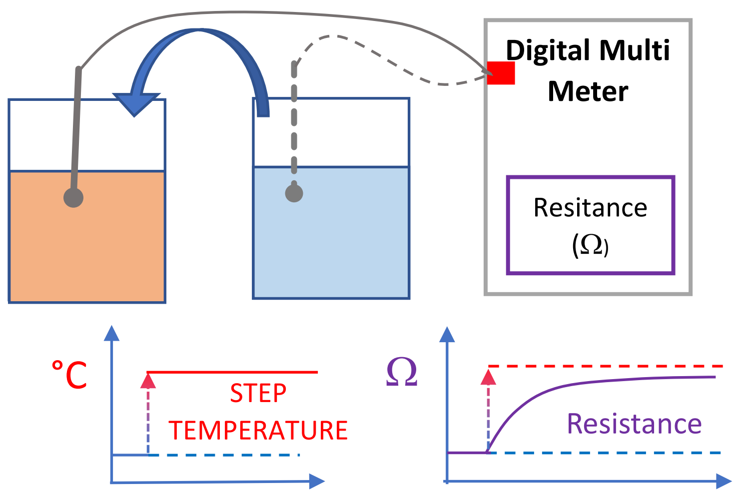
$$Y(s) \Rightarrow L^{-1}(s) \Rightarrow \frac{y(t)}{\Delta_0 A} = 1 - e^{-\frac{t}{\tau}}$$

Inverse Laplace transform of $\mathrm{Y}(s)$ give the dynamic response $\mathrm{y}(t)$ in time domain.

Test results $\tilde{y}(t)$ can be fitted with equation $\mathrm{y}(t)$ to evaluate the parameter τ.

// Thermal energy meter – Parameter estimation

Temperature sensor - step response

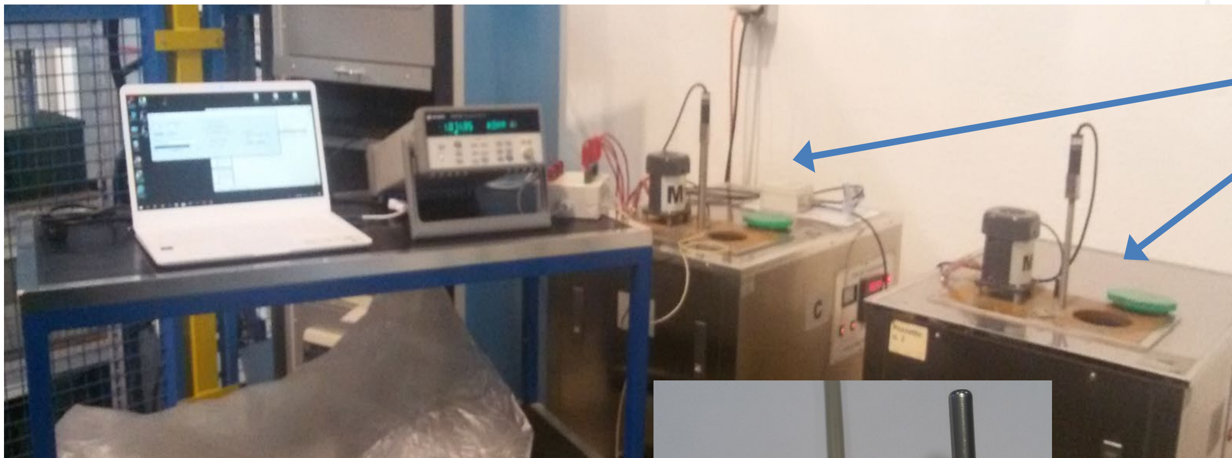


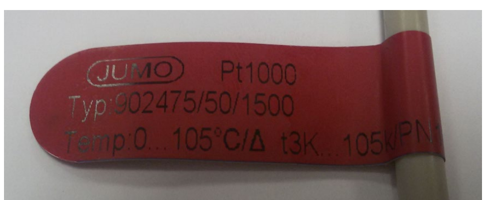
$$\frac{R(t) - R_0}{\Delta R} = 1 - e^{-\frac{t}{\tau_\theta}}$$

Test results $R(t)$ can be fitted to evaluate the parameter τ_θ.

// Thermal energy meter – Parameter estimation

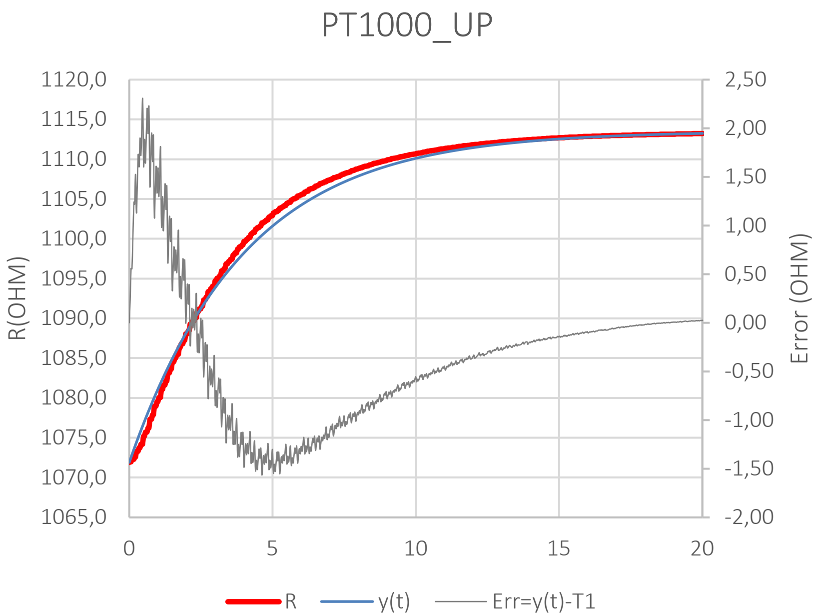
Temperature sensor - step response

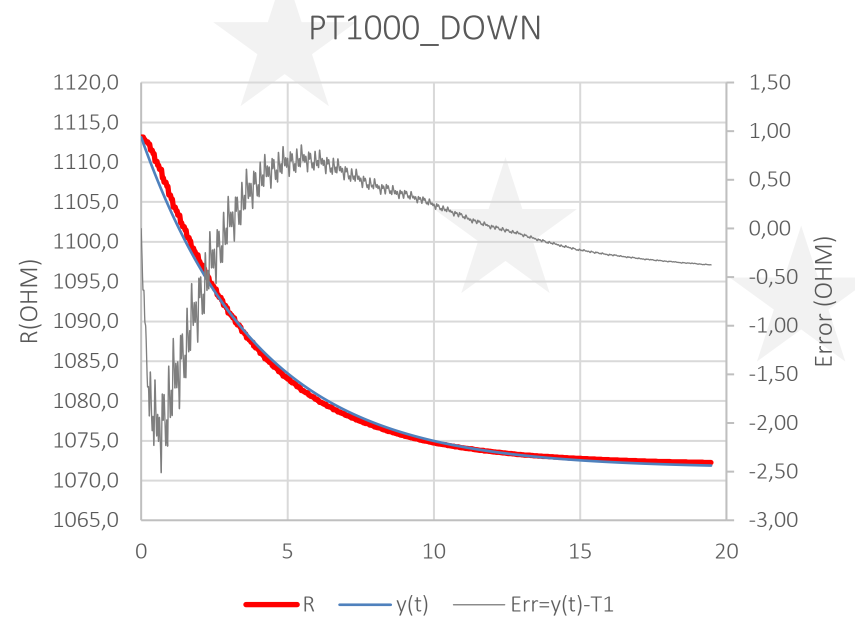




// Thermal energy meter – Parameter estimation

Temperature sensor - step response



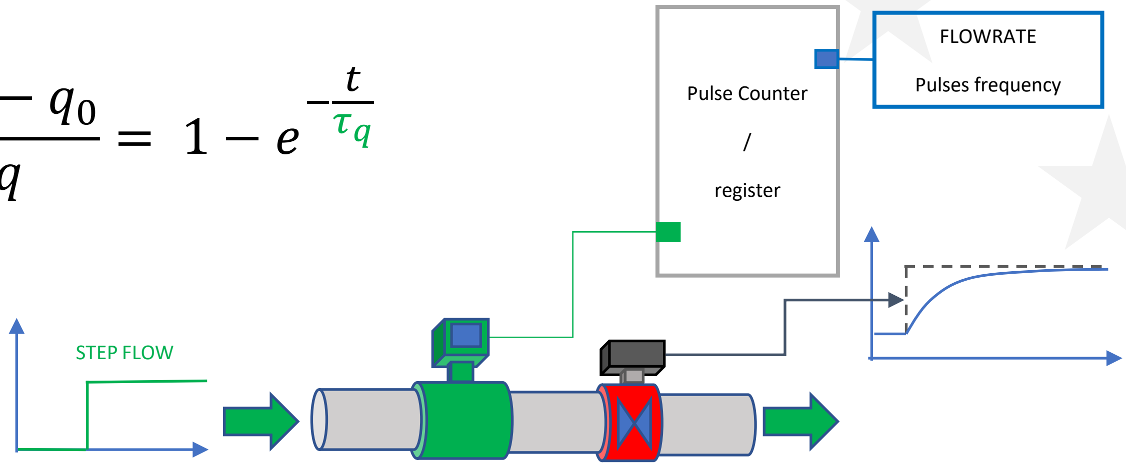


θ_{hot}=28,75°C, θ_{cold}=18,07°C, $\Delta\theta$=10,68°C , ΔR=41,63Ω , τ_{θ}=4s

// Thermal energy meter – Parameter estimation

Flow sensor - step response

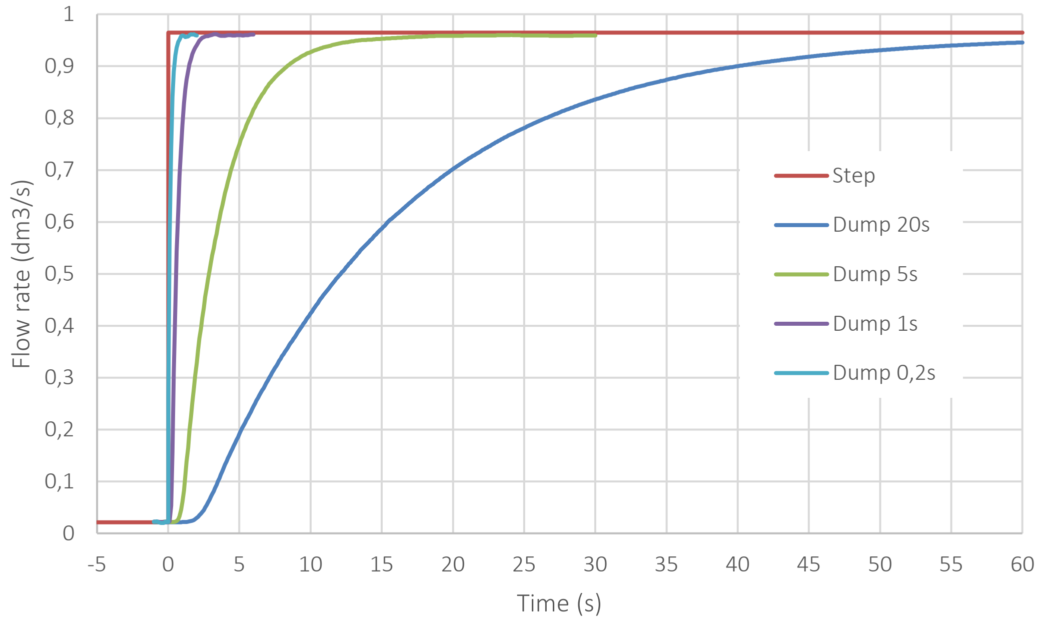
$$\frac{q(t) - q_0}{\Delta q} = 1 - e^{-\frac{t}{\tau_q}}$$



Test results $q(t)$ can be fitted to evaluate the parameter τ_q.

// Thermal energy meter – Parameter estimation

Flow sensor - step response



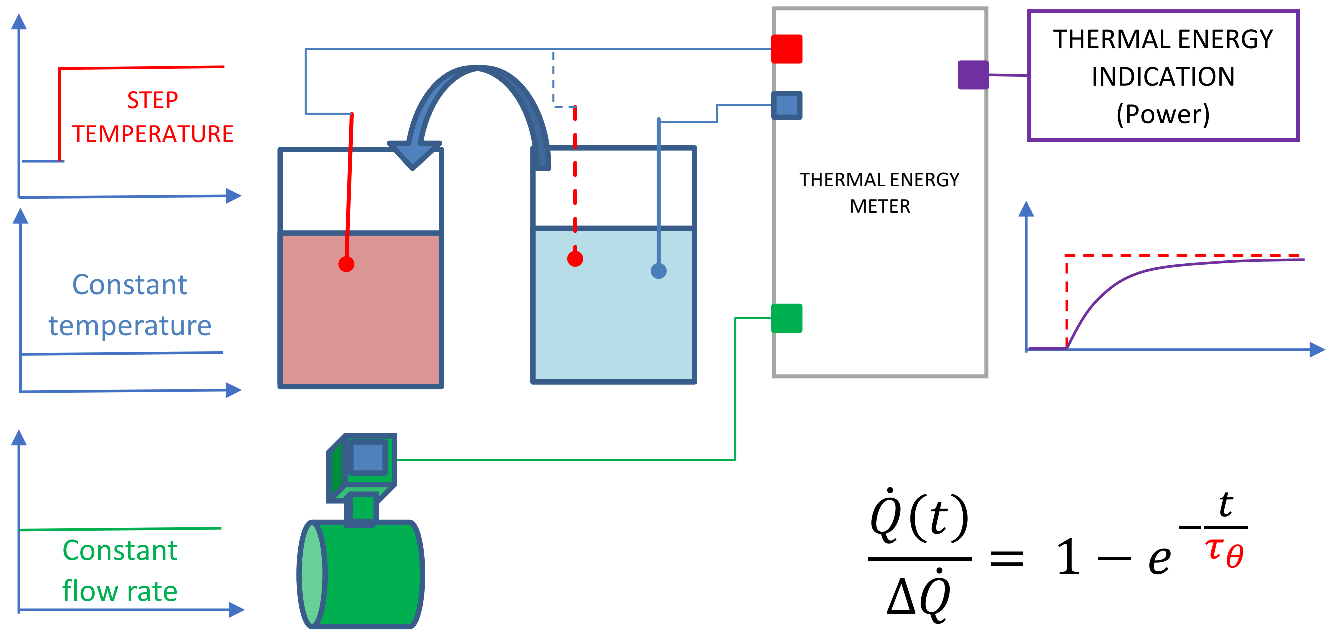
Flow meter
(sensor simulator)

Flow sensor parameter: τ_q

Dumping software selectable. Example from τ_q=0,2s up to τ_q=20s

// Thermal energy meter – Parameter estimation

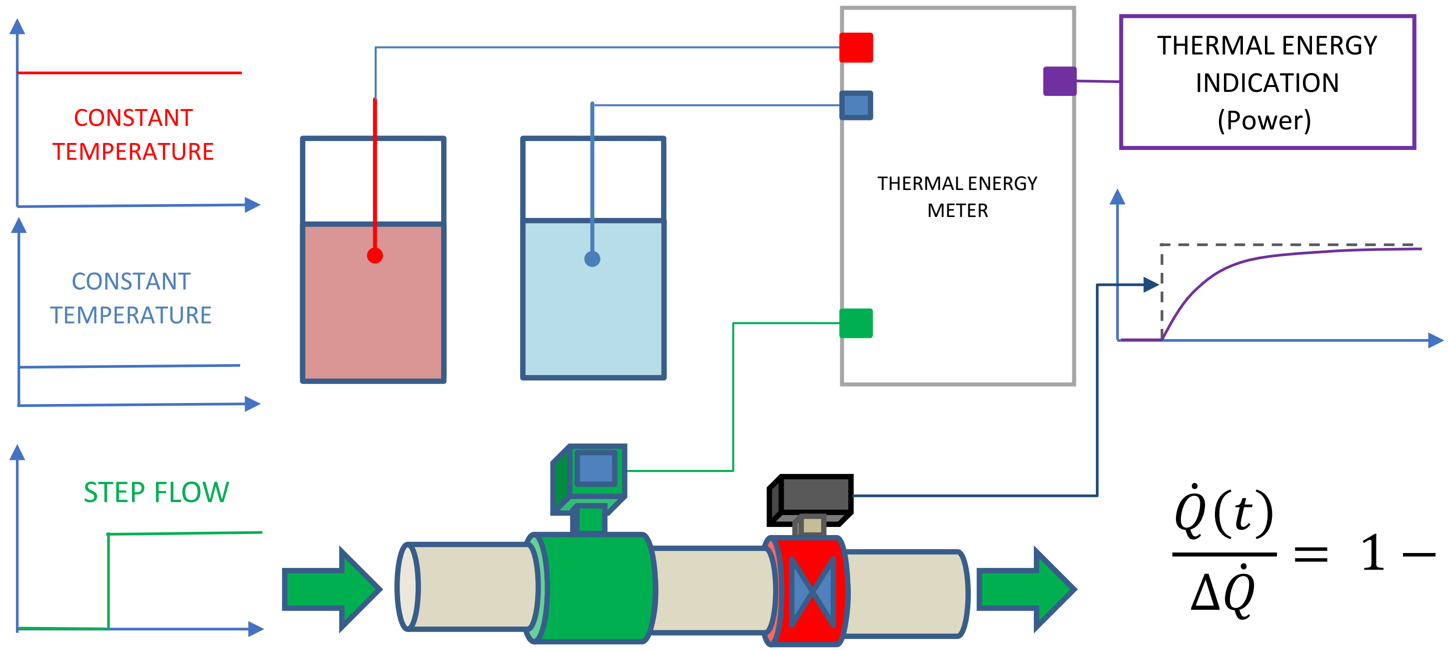
Complete thermal energy meter – temperature step response



$$\frac{\dot{Q}(t)}{\Delta\dot{Q}} = 1 - e^{-\frac{t}{\tau_\theta}}$$

// Thermal energy meter – Parameter estimation

Complete thermal energy meter – flow step response



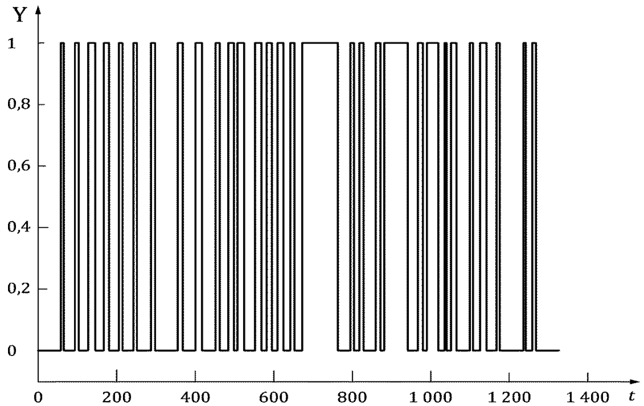
$$\frac{\dot{Q}(t)}{\Delta\dot{Q}} = 1 - e^{-\frac{t}{\tau_q}}$$

05

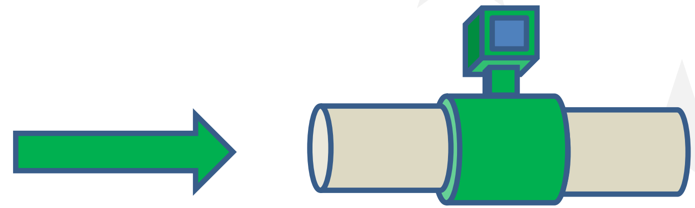
Thermal Energy Meter – Fast response meters: a new point of view about the problem

// Fast response meters: a new point of view about the problem

Flow sensor test – actual approach EN 1434-4:2020



$$q(t) = K_{factor} f(t)_{pulses}$$



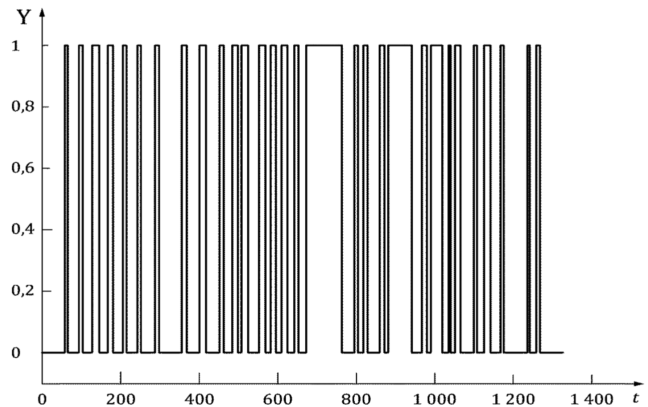
$$Vref = \int Y(t)dt$$

$$V = \int q(t)dt$$

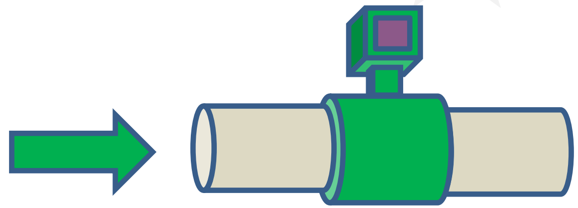
$$error = \frac{V - Vref}{Vref} \times 100 \leq MPE\ ?$$

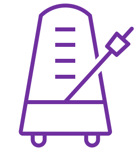
// Fast response meters: a new point of view about the problem

Flow sensor test – actual approach EN 1434-4:2020



$$q(t_k) = q(k\Delta t)$$





Δt : SAMPLING TIME

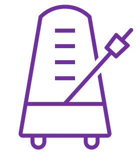
$$Vref = \int Y(t)dt$$

$$V = \Delta t \times \sum q(k\Delta t)$$

$$error = f(\Delta t)$$

// Fast response meters: a new point of view about the problem

NEW APPROACH: How much fast I need to sample to avoid infomation loss?



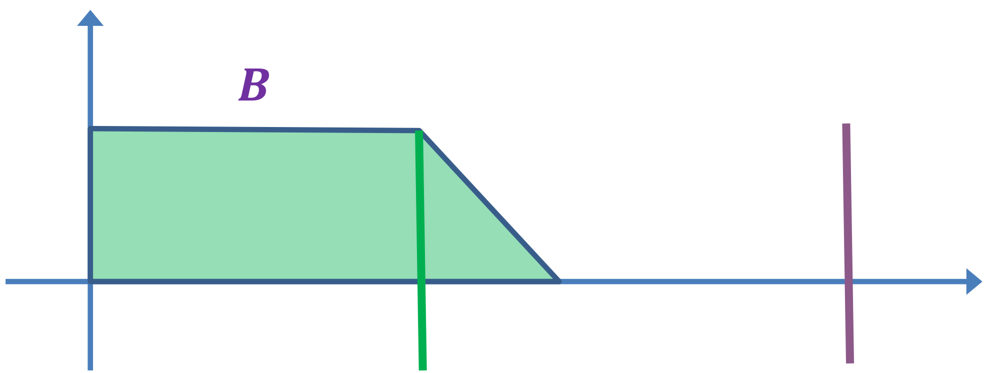
Δt : SAMPLING TIME

The Nyquist–Shannon sampling theorem is a theorem which:

- serves as a fundamental bridge between continuous-time signals and discrete-time signals.
- establishes a sufficient condition for a sample rate that permits a discrete sequence of samples to capture all the information from a continuous-time signal of finite bandwidth.

The **Nyquist–Shannon sampling theorem** : for a given sample rate $f_s = \frac{1}{\Delta t}$,perfect reconstruction is guaranteed possible for a bandlimit $B < \frac{f_s}{2}$

Δt : SAMPLING TIME

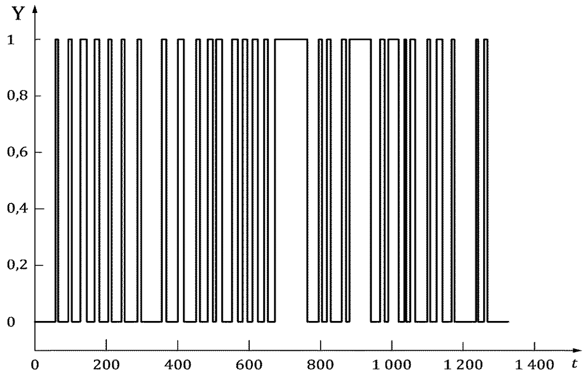


$$f_q = \frac{1}{2\pi\tau_q} < \frac{f_s}{2}$$

$$f_s = \frac{1}{\Delta t}$$

$$\tau_q = \frac{\Delta t}{\pi}$$

// Fast response meters: a new point of view about the problem



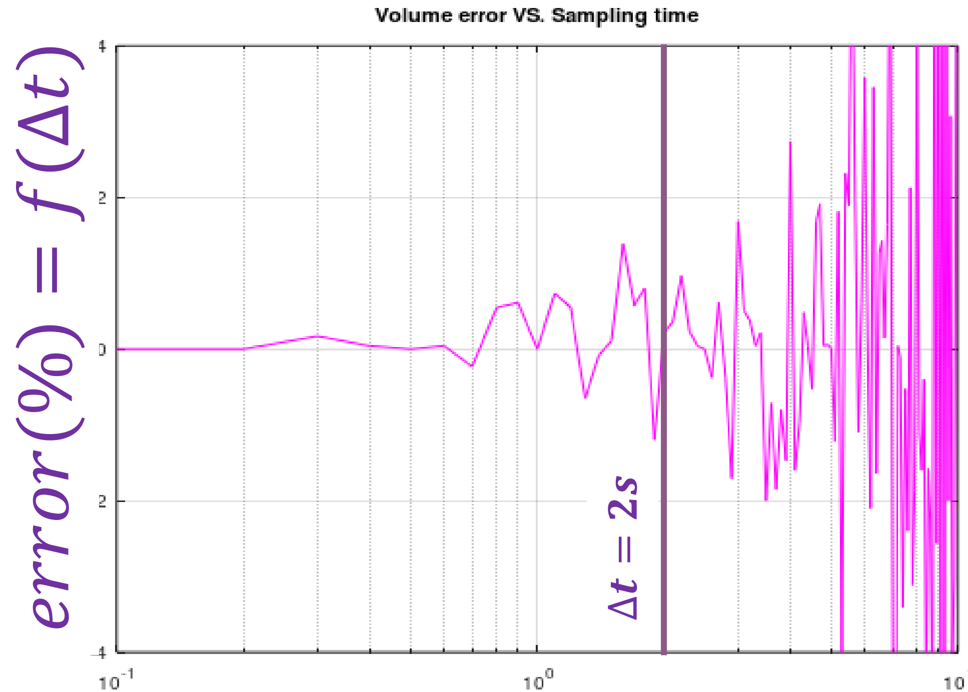
$$V = \Delta t \times \sum q(k\Delta t)$$

prEN 1434-1:2020 (E) Annex C

Volume sampling time interval:

≤ 2 Seconds

(non-residential buildings and family houses)



Δt : SAMPLING TIME (s)

// Fast response meters: a new point of view about the problem

prEN 1434-1:2020 (E) Annex C

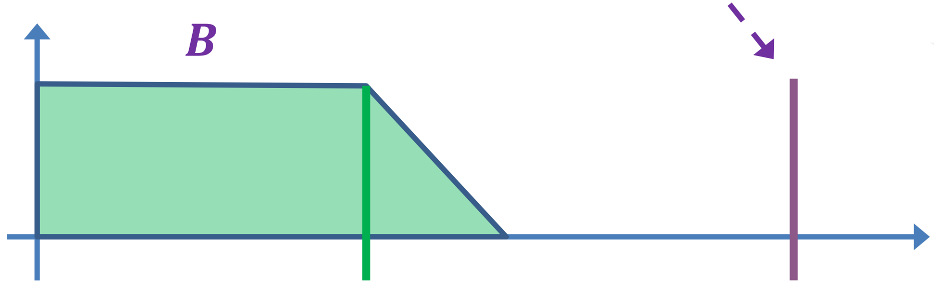
Volume sampling time interval:

$$\Delta t \leq 2s$$

(non-residential buildings and family houses)

$$f_{s,min} = \frac{1}{\Delta t} = 0,5Hz$$

Δt : SAMPLING TIME

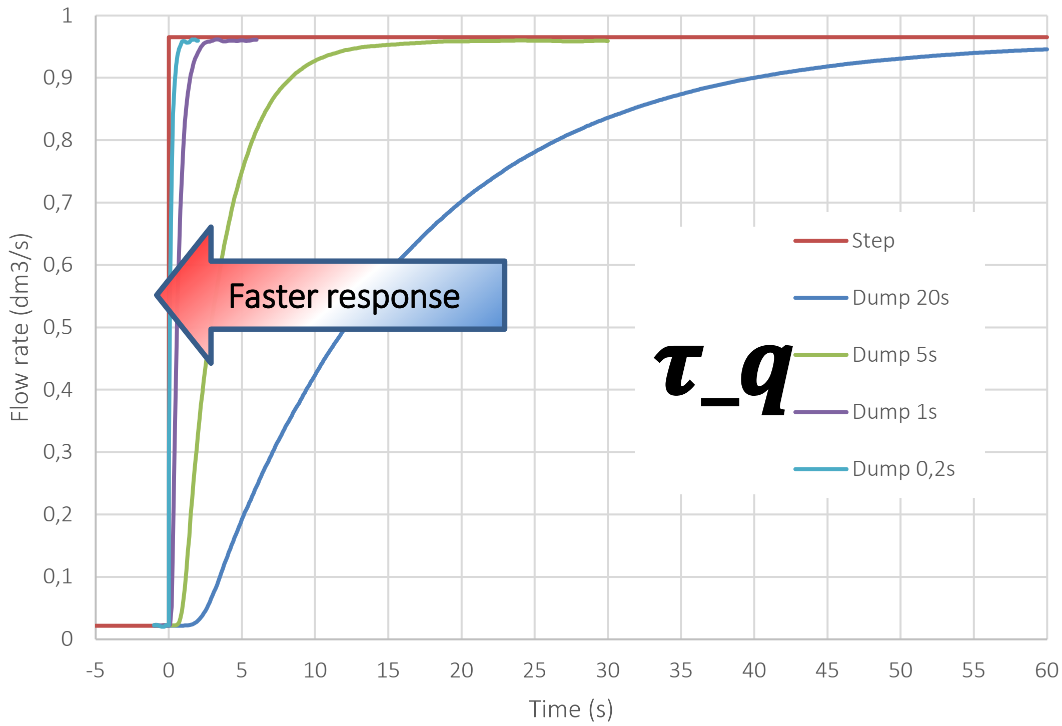


Nyquist–Shannon sampling theorem

$$f_{q,max} = \frac{1}{2\pi\tau_q} < \frac{f_{s,min}}{2} = 0,25Hz$$

$$\tau_q > \frac{\Delta t}{\pi} = \frac{2}{\pi} = 0,64s$$

// Fast response meters: a new point of view about the problem



τ_q	$\Delta t, max$
(s)	(s)
0,10	0,31
0,20	0,63
0,40	1,26
0,64	**2,0**
0,80	2,5
1	3,1
2	6,3
4	13
8	25
16	50
32	100

Δt : SAMPLING TIME

06

Thermal Energy Meter – Conclusion

// Thermal energy meter – Conclusion

- Dynamic model of 1°st order is a useful instrument
- is <u>**simple**</u> and <u>**robust**</u> and need only two parameters : (τ_θ, τ_q)
- test methods for estimate (τ_θ, τ_q) parameters are available
- Dimensional homogeneity on quantity involved:
  - τ_θ time constant for temperature measurement
  - τ_q time constant for volume measurement
- Is possible to define a dynamic response classification <u>and fast response requirements</u> based on (τ_θ, τ_q)

  (Note: sampling time became a matter of manufacturer not the notified body!)

THANK YOU FOR YOUR ATTENTION

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